Corneal Asphericity Change after Excimer Laser Hyperopic Surgery: Theoretical Effects on Corneal Profiles and Corresponding Zernike Expansions

Damien Gatinel, Jacques Malet, Thanh Hoang-Xuan, and Dimitri T. Azar

PURPOSE. To determine the theoretical relationships between the changes in corneal paraxial power, asphericity, and the corresponding Zernike polynomial expansion after conventional and customized excimer laser correction of hyperopia.

METHODS. The initial corneal profile was modeled as a conic section of apical radius of curvature \( R_1 \) and asphericity \( Q_1 \). The theoretical value of the postoperative apical radius of curvature \( R_2 \) was computed by using a paraxial formula from the value of \( R_1 \) and hyperopic defocus \( D \). The postoperative asphericity \( Q_2 \) of the corneal surface was computed within the optical zone of diameter \( S \) after the delivery of a Munnerlyn-based profile of ablation for hyperopia using conic section-fitting and minimization of the squared residuals. These calculations were repeated for different values of defocus, initial apical radius of curvature, and asphericity. Taylor series expansions were also used to provide an approximation aimed at predicting change in asphericity. The coefficients of a Zernike polynomial expansion of the rotationally symmetrical corneal profile (defocus \( C_{60} \), spherical aberration \( C_{40} \), secondary spherical aberration \( C_{60} \)) were also computed, by using scalar products applied to the considered corneal profile modeled as a conic section and were expressed as a function of both its apical radius and asphericity. This allowed approximation of the variations of the Zernike polynomial expansion of the corneal profiles by subtracting the postoperative coefficient weighting a particular aberration from that of the preoperative one in different theoretical situations, after both conventional and customized hyperopia treatments aimed at controlling the postoperative corneal asphericity and delivered over a normalized pupil diameter.

RESULTS. Conical least-squares fitting was unambiguous, allowing approximation of the postoperative corneal profile as a conic section of apical radius \( R_2 \). After a Munnerlyn-based hyperopia treatment, the sign of the asphericity of this profile remains theoretically unchanged, but its value decreased for initially oblate and increased for initially prolate corneas, respectively. A similar trend was noted with the approximation obtained by the Taylor series expansion. The alteration of the apical radius and/or of the asphericity of the corneal surface resulted in variations of both the corneal profile Zernike coefficients \( C_{60} \) and \( C_{40} \). The former was essentially dependent on the variation of the apical radius and the latter essentially on the variation of both apical radius and asphericity.

CONCLUSIONS. Conventional and customized profiles of ablation for hyperopia alter the postoperative corneal asphericity and the Zernike coefficients of the corneal profile. The results of this study may be useful in the interpretation of the postoperative variations of the corneal profile and their impact on corneal wavefront expansion variations after both conventional and customized profiles of ablation. (Invest Ophthalmol Vis Sci. 2004;45:1349–1359) DOI:10.1167/iovs.03-0753

Surgical correction of hyperopia with the excimer laser involves steepening of the central anterior corneal surface and creation of a peripheral annular blend zone. Recent studies have shown increased negative asphericity after photorefractive keratectomy (PRK) and laser in situ keratomileusis (LASIK) for hyperopia. The changes after PRK, calculated from Zernike polynomial coefficients, correlate with the magnitude of treatment. Spherical aberration increases dramatically with increasing pupil diameters. This may account for the reported loss of optical quality in low-luminance situations and for the symptoms of glare after excimer laser surgery for hyperopia.²,³

One objective was to determine theoretical relationships among corneal paraxial power, corneal asphericity, and corneal shape variations (described through an expansion of Zernike terms) after conventional and customized hyperopia treatments. Because of the combination of corneal steepening within the optical zone and corneal flattening in the surrounding annular blend zone, the contributions of these two zones to the increased negative asphericity after surgery for hyperopia may be difficult to assess.

Our approach involved determining the theoretical effects of spherically based (conventional) and customized ablations on the corneal shape changes, through calculating their effect on corneal asphericity. The latter was primarily achieved by using a method of conic section-fitting to determine postoperative asphericity.² We also developed an analytical approach, using scalar products to convert the aspheric conical equations for the pre- and postoperative surfaces to the corresponding Zernike terms. This allowed us to evaluate the influence of these treatments on corneal profile spherical \( Z_{60} \) variations for various magnitudes of hyperopic defocus \( Z_2 \) corrections.

METHODS

Effect of Conventional (Spherical) Hyperopia Ablation on Postoperative Corneal Asphericity and Corresponding Zernike Polynomial Expansions

Determination of Corneal Asphericity after Conventional (Noncustomized) Hyperopia Ablation. Current excimer laser ablations for treating hyperopia rely on the work of Munner-
lyn et al., in which the initial and final corneal surfaces are assumed to be spherical \((Q_1 = Q_2 = 0)\). This allows calculation of the ablation profile by the general formula (Fig. 1)

\[
dZ = R_1 - R_2 + \sqrt{R_2^2 - (x^2 + y^2)} - \sqrt{R_1^2 - (x^2 + y^2)} \tag{1}
\]

where \(dZ\) expresses the depth of tissue removal as a function of the distance \(\sqrt{x^2 + y^2}\) from the center of an optical zone diameter of \(S\), when \(R_1\) and \(R_2\) are the initial and final corneal anterior radii of curvature, respectively. The power of the removed lenticule \((D)\) corresponds to the intended refractive change and is related to \(R_1\), \(R_2\), and the index of refraction \((n)\). \(R_2\) is calculated from the intended magnitude of treatment \(D\) by the paraxial formula

\[
D = (n - 1) \cdot \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \tag{2}
\]

Our preliminary approach to determine the postoperative asphericity of the aspheric corneal surface treated with conventional hyperopia ablations (Figs. 2, 3) was conic section-fitting by minimizing the sum of squared residuals (Appendix A).

Referring to a geometric model and using a mathematical procedure as defined in Appendix A, we used a conic section-fitting method to determine the postoperative asphericity of the cornea within the optical zone after paraxial laser excimer treatment for hyperopia. The Munnerlyn-based profile of ablation was then subtracted from the conic section modeling the preoperative corneal surface. The analytical expression of the resultant curve did not correspond to an obvious conic section. The postoperative apical radius of curvature \(R_c\) was calculated as the radius of the circle at the apex of the curve and equal to \(R_2\) (Appendix A, equations 13 and 15). To determine the postoperative asphericity of the curve, \(Q_c\), the conical least-squares fitting was then applied iteratively to the function until convergence was achieved (Fig. 2). These calculations were performed for a range of different dioptric treatments, initial asphericities, and radii of curvature, to estimate postoperative asphericity.
The change of asphericity, $dQ$, after conventional hyperopia treatment was also approximated as described in Appendix A. The Munnerlyn profile of a spherically based treatment was mathematically equalized to the difference between two aspheric surfaces with asphericity factors $Q_1$ and $Q_2$. Taylor expansions were used to approximate the change of asphericity ($dQ$) as

$$dQ = -8D R_1 Q_1$$  \((3)\)

The values obtained from the approximations in equation 3 were compared to those obtained by the method of least-squares fitting to determine $Q_2$, (Appendix A).

**Determination of Corneal Profile Changes after Spherical Treatment, Using Zernike Polynomial Expansion.** Approximation of the corneal profile as described by a conic section of apical radius $R$ and asphericity $Q$ can be converted in a Zernike polynomial expansion over a zone of diameter $S$ by using scalar parameters. This allows expression of the corneal profile as a linear combination of Zernike polynomials ($Z_n^m$), each weighted by a coefficient $C_n^m$ that is a function of $R$ and $Q$. This method is detailed in Appendix B.

Because of the symmetrical property of our model, the rotationally invariant polynomials ($m = 0$) have to be computed. We calculated the coefficient for each polynomial up to the 10th order (Appendix B). A normalized pupil of radius $S/2$ was considered. Because of the normalizing constraint, the apical radius $r$ used for analysis was determined as

$$r = 2R/S$$  \((4)\)

Equations 5a, 5b, and 5c show $C_2^0$, $C_4^0$, and $C_6^0$, the coefficients of the second-, fourth-, and sixth-order rotationally invariant polynomials, respectively,

$$C_2^0 = \frac{\sqrt{3}(Q + 1)}{12r} + \frac{3 \sqrt{3}(Q + 1)^3}{520r^3} + \frac{\sqrt{3}(Q + 1)^5}{192r^5} + \frac{3 \sqrt{3}(Q + 1)^7}{1536r^7} + \frac{\sqrt{3}(Q + 1)^9}{5072r^9} \tag{5a}$$

$$C_4^0 = \frac{\sqrt{3}(Q + 1)}{240r} + \frac{\sqrt{3}(Q + 1)^3}{520r^3} + \frac{\sqrt{3}(Q + 1)^5}{192r^5} + \frac{3 \sqrt{3}(Q + 1)^7}{1536r^7} + \frac{3 \sqrt{3}(Q + 1)^9}{5072r^9} \tag{5b}$$

$$C_6^0 = \frac{\sqrt{3}(Q + 1)}{2240r^3} + \frac{\sqrt{3}(Q + 1)^3}{48r^3} + \frac{\sqrt{3}(Q + 1)^5}{9216r^5} + \frac{\sqrt{3}(Q + 1)^7}{48r^7} + \frac{\sqrt{3}(Q + 1)^9}{9216r^9} \tag{5c}$$

Equations 5a–c show that these coefficients aimed at describing the corneal profile modeled as a conic section over a given diameter are proportional, not only to the value of the asphericity $Q$, but also to the reciprocal of the apical radius of curvature (i.e., proportional to the paraxial power).

The initial and final corneal profiles were modeled as conic sections of apical radii $R_1$ and $R_2$, and initial asphericity $Q_1$ and $Q_2$, respectively. $R_2$ was computed with a paraxial formula. $Q_2$ was obtained by the method of minimizing the sum of squared residuals (Appendix A). The Zernike coefficients $C_2^0$, $C_4^0$, and $C_6^0$ were computed for the preoperative and postoperative corneal profiles. The variation for each of the coefficients $\Delta C_n^0$ was computed as the difference between the postoperative ($C_n^0$) and the preoperative ($C_n^0$) values

$$\Delta C_n^0 = C_n^0 - C_n^0 \tag{6}$$

The variations of the coefficients $C_n^0$ and $C_n^0$ (i.e., $\Delta C_n^0$ and $\Delta C_n^0$) were investigated in various theoretical situations. The effects of the magnitude of paraxial treatment, initial corneal asphericity, and initial apical radius of curvature were calculated for different values within the clinical range.

**Effect of Customized Hyperopia Ablation on Postoperative Corneal Asphericity and Corresponding Zernike Polynomial Expansions**

**Determination of an Aspheric Profile of Ablation to Treat Hyperopia.** The pre- and postoperative corneal profiles were modeled as conic sections of apical radii $R_1$ and $R_2$ and asphericity factors $Q_1$ and $Q_2$, respectively. $R_2$ was calculated from the intended magnitude of treatment $D$ by the paraxial formula (equation 2).

The pattern of ablation was calculated as the difference in sagittal height for each point of the initial and final surfaces, when they coincide at the center of the optical zone of diameter $S$ at the outer margin of the optical zone. $S/2 = \sqrt{S^2 + 3r^2}$. The ablation pattern corresponds to the material removed between two aspheric surfaces whose curvature difference results in the targeted change in apical power and asphericity.

Using finite analysis, the aspheric ablation profile (Fig. 3, shaded area) was given by

$$dZ = (x_2 - x_1) = -R_2 + \sqrt{\left(\frac{R_1^2 - (1 + Q_2)\frac{S^2}{2}}{1 + Q_1}\right)^2}$$

$$-R_1 + \sqrt{\left(\frac{R_1^2 - (1 + Q_1)\frac{S^2}{2}}{1 + Q_1}\right)^2} \tag{7}$$

Using the Taylor series expansion up to the second order, the aspheric profile of ablation was also approximated as described previously.

$$dZ = \frac{S^2 D}{3} + \left(\frac{3S^2 Q_2 D}{16r^2}\right) \left(\frac{S^2 D}{3}\right) + \left(\frac{3S Q_2 D}{16r^2}\right) \left(\frac{S^2 D}{3}\right) + \left(\frac{dQ S^2 D}{3}\right) \tag{8}$$

**Variations of the Zernike Coefficients $C_2^0$ and $C_4^0$ after Customized Treatment.** The variations of the coefficients $C_2^0$ and $C_4^0$ were investigated for an ablation pattern that would allow control of the postoperative corneal asphericity within the optical zone. The effect of tuning the corneal asphericity with no modification of the paraxial power change was studied. Conversely, the variation of the coefficient was also computed for a profile that would preserve the initial corneal asphericity while changing the paraxial power within the optical zone.

**Table 1. Influence of Initial Apical Radius of Curvature ($R_1$) on Final Asphericity for an Initially Oblate Cornea ($Q_1 = 0.4$)**

<table>
<thead>
<tr>
<th>Magnitude of Treatment ($D$)</th>
<th>$R_1 = 6.8$ mm</th>
<th>$R_1 = 7.8$ mm</th>
<th>$R_1 = 8.8$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>1</td>
<td>0.38</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>0.31</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>6</td>
<td>0.28</td>
<td>0.27</td>
<td>0.26</td>
</tr>
</tbody>
</table>
**RESULTS**

**Effect of Conventional (Spherical) Hyperopia Ablation on Postoperative Corneal Asphericity and Corresponding Zernike Polynomial Expansions**

Determination of Corneal Asphericity after Conventional (Noncustomized) Hyperopia Ablation. Conical least-squares fitting led to convergence by minimizing the sum of the offsets, and the determination of $Q_2$ was unambiguous (Fig. 2).

Tables 1 and 2 show the minimal influence of the initial apical radius of curvature for three different values, 6.8, 7.8, and 8.8 mm, for initially oblate and prolate corneas. In both situations, there was no significant influence of the initial apical radius of curvature on postoperative asphericity. Figure 4 represents the theoretically predicted postoperative asphericity after hyperopic treatment based on initial corneal asphericity. Figure 4 allows comparison of the two methods of estimating the variation in asphericity induced by paraxial hyperopia treatments conforming to the Munnerlyn formula—namely, minimization of the sum of the squared residual, MSR, and the approximation of equation 3.

**Determination of Corneal Profile Changes after Spherical Treatment, by Using Zernike Polynomial Expansion.** Table 3 shows the theoretical variation of the value of the coefficients $C_2^0$, $C_4^0$, and $C_6^0$ after delivery of a spherical profile of ablation of magnitude ranging from +1 to +6 D over a 6-mm diameter optical zone. The initial corneal apical radius and asphericity were 7.8 mm and $-0.2$, respectively. The variation of the coefficients increased in a linear fashion with the magnitude of treatment. Each diopter of treated hyperopia led to a variation of approximately 1.3 $\mu$m of the $C_2^0$ coefficients.

The theoretical influence of asphericity and apical radius of curvature of the initial corneal surface on $C_2^0$ and $C_4^0$ were investigated; results are shown in Figure 5. The more oblate, and the steeper the initial surface, the more substantial was the variation in $C_2^0$ and $C_4^0$ for the same magnitude of paraxial hyperopia treatment.

**Effect of Customized Hyperopia Ablation on Postoperative Corneal Asphericity and Corresponding Zernike Polynomial Expansions**

Determination of an Aspheric Profile of Ablation to Treat Hyperopia. The accuracy of our approximation using the Taylor series of expansion (equation 8) was checked in

### Table 2. Influence of Initial Apical Radius of Curvature ($R_1$) on Final Asphericity for an Initially Prolate Cornea ($Q_1 = -0.60$)

<table>
<thead>
<tr>
<th>Treatment Magnitude ($D$)</th>
<th>$R_1 = 6.8$ mm</th>
<th>$R_1 = 7.8$ mm</th>
<th>$R_1 = 8.8$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-0.60$</td>
<td>$-0.60$</td>
<td>$-0.60$</td>
</tr>
<tr>
<td>1</td>
<td>$-0.56$</td>
<td>$-0.56$</td>
<td>$-0.56$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.55$</td>
<td>$-0.52$</td>
<td>$-0.52$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.50$</td>
<td>$-0.49$</td>
<td>$-0.48$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.47$</td>
<td>$-0.46$</td>
<td>$-0.45$</td>
</tr>
<tr>
<td>5</td>
<td>$-0.44$</td>
<td>$-0.43$</td>
<td>$-0.42$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.42$</td>
<td>$-0.40$</td>
<td>$-0.39$</td>
</tr>
</tbody>
</table>

### Table 3. Change of $C_2^0$, $C_4^0$, and $C_6^0$ after Spherical Ablations

<table>
<thead>
<tr>
<th>Treatment Magnitude ($D$)</th>
<th>$\Delta C_2^0$</th>
<th>$\Delta C_4^0$</th>
<th>$\Delta C_6^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>1.291</td>
<td>0.038</td>
<td>0.0011</td>
</tr>
<tr>
<td>+2</td>
<td>2.617</td>
<td>0.087</td>
<td>0.0026</td>
</tr>
<tr>
<td>+3</td>
<td>3.927</td>
<td>0.131</td>
<td>0.0040</td>
</tr>
<tr>
<td>+4</td>
<td>5.247</td>
<td>0.178</td>
<td>0.0055</td>
</tr>
<tr>
<td>+5</td>
<td>6.577</td>
<td>0.228</td>
<td>0.0075</td>
</tr>
<tr>
<td>+6</td>
<td>7.884</td>
<td>0.271</td>
<td>0.0088</td>
</tr>
</tbody>
</table>

Data are expressed in micrometers. Calculations are based on $R_1 = 7.8$ mm and $Q_1 = -0.2$. The computed values of $C_6^0$ were very low and possibly not clinically significant.

The theoretical influence of asphericity and apical radius of curvature of the initial corneal surface on $C_2^0$ and $C_4^0$ were investigated; results are shown in Figure 5. The more oblate, and the steeper the initial surface, the more substantial was the variation in $C_2^0$ and $C_4^0$ for the same magnitude of paraxial hyperopia treatment.
The profile of ablation given by finite analysis (equation 7) and Taylor approximations (equation 8) are plotted on the same graph for various magnitudes of treatment, initial asphericities, and intended variations in asphericity. When the intended asphericity is more oblate ($dQ_{1022}/H_{0}$), equation 8 tends to slightly underestimate the depth of ablation given by the finite analysis (equation 7). When the intended asphericity is more prolate ($dQ_{1021}/H_{0}$), equation 8 tends to slightly overestimate the depth of ablation given by the finite analysis.

Variations of the Zernike Coefficients $C_{20}$ and $C_{40}$ after Customized Treatment. The influence of the tuning of the postoperative asphericity over the optical zone without changing its apical power was investigated for an initially oblate and an initially prolate corneal profile (Fig. 7). When the asphericity is decreased (increased prolateness), both $\Delta C_{20}$ and $\Delta C_{40}$ are negative (decrease of the corresponding variance of the corneal Zernike expansion). When the asphericity is increased (increased oblateness), both $\Delta C_{20}$ and $\Delta C_{40}$ are positive (increase of the variance of the corneal Zernike expansion).

The influence of tuning the postoperative apical radius (without changing the asphericity of the corneal profile) over the optical zone was also studied by selective variation of the paraxial optical power (Fig. 8). The increase in $\Delta C_{20}$ varied linearly with the magnitude of the intended change in apical optical power and was much greater than that of $\Delta C_{40}$.

DISCUSSION

We were able to estimate the corresponding theoretical changes in the coefficients of the rotationally symmetric even-order Zernike polynomial of the corneal shape (defocus: $Z_{20}^{0}$, spherical aberration $Z_{40}^{0}$, and $Z_{60}^{0}$) after laser surgery for non-astigmatic hyperopia. The spherically based hyperopia treatments induced theoretical variations in all the tested rotationally symmetric even-order coefficients. The tendency toward spherical approximation ($Q_{2} < Q_{1}$) was accompanied by an increase in variation in $C_{0}^{0}$ for both initially oblate and initially prolate corneas. For customized treatments, the variations in
preoperative and postoperative apical radius induced variations in both $C_4^0$ and $C_2^0$. We observed the same tendency with regard to variations in preoperative and postoperative asphericity. However, the variations in $C_4^0$ and $C_2^0$ were both negative when the asphericity decreased (increased prolateness, decreased oblateness), and both positive when asphericity increased (decreased prolateness, increased oblateness).

These data can be explained by the fact that the reference surface for cornea-derived wavefront aberrations is the aberration-free prolate surface of a parabola, rather than the surface of a sphere. For any conic section including the circle (spherical surface: $Q = 0$), our calculations show that $C_4^0$ increased with a decreasing apical radius of curvature. It is thus not surprising that after correction of hyperopia, the intended decrease in the apical radius of curvature results in increased $C_4^0$, despite decreased $Q_2$: in the case of initial oblateness. A model based on the subtraction from a parabolic surface of another parabolic surface of different apical curvature is necessary, to allow the theoretical preservation of all the Zernike coefficients, save $Z_2^0$.

Consequently, the conventional treatments based on spherical models are expected to modify all the coefficients of rotationally symmetrical Zernike polynomials. Our data show that in the case of hyperopia, the theoretical variation in each coefficient is proportional to the magnitude of treatment.

After conventional excimer laser surgery for myopia, positive spherical aberration is dramatically increased. This is commonly attributed to the oblate change in corneal asphericity after surgery. The aberration theory predicts that for a rotationally symmetric optical system, the power error induced by spherical aberration can be expressed as an even-order power function in ray height. Most eyes are thought to suffer from positive spherical aberration when unaccommodated. The natural prolate shape of the cornea is thought to complement the crystalline lens in reducing the positive spherical aberration of the eye. Profound modification of corneal asphericity occurs after conventional refractive surgery aimed at correcting defocus, but is associated with loss of contrast sensitivity in scotopic conditions.

We have investigated the measured changes in corneal asphericity after excimer laser surgery for hyperopia and observed a tendency of increased postoperative prolateness after conventional LASIK for hyperopia with initially prolate corneas. There was a positive relationship between the amount of intended correction and the degree of postoperative prolateness. A study by Oliver et al. disclosed a large increase in corneal spherical aberration for large pupil diameter after conventional photorefractive keratectomy using an excimer laser (Apex Plus; Summit Technology, Boston, MA). This increase was more dramatic than after surgery for myopia and was attributed to an excessive flattening of the peripheral cornea. Based on these studies, there may be a discrepancy between the predicted and observed postoperative corneal asphericity after conventional refractive excimer laser ablation of hyperopia, because our calculations disclose that for an initially prolate cornea, the postoperative corneal profile could be approximated by a conic section and should be less prolate. We used two different methods and obtained similar results: After conventional excimer laser treatment, the postoperative
variation in asphericity of the corneal profile can be adequately approximated by a linear relationship and is proportional to the intended magnitude of treatment, preoperative apical radius of curvature, and asphericity.

The discrepancy between predicted and observed asphericity may be the result of factors similar to those invoked to account for the discrepancy between predicted and observed corneal asphericity after surgery for myopia. First, the nomograms of most excimer lasers are proprietary, and adherence to the paraxial model of Munnerlyn cannot be established. Second, profiles of ablation for hyperopia deliver the maximum number of laser pulses at the periphery of the optical zone (where the beam is not orthogonal to the corneal surface), and any variation of the applied fluence over this peripheral area may change the tissue ablation rate and modify the intended surface remodeling. Third, the theoretical shape of the transition zone carved at the corneal midperiphery is also proprietary. It is possible that specular topography, such as that used by Oliver et al. is inaccurate for measuring the curvature at the transition zone. This could decrease the accuracy of calculating the local radius of curvature, the asphericity, and/or the corneal optical aberrations.

A fourth possible explanation is the biomechanical change in the cornea. Roberts and Dupps have proposed a model to explain the unexpected central flattening after myopia or plano ablations. They hypothesized that central ablation during PRK, phototherapeutic keratectomy (PTK), and LASIK causes a circumferential severing of corneal lamellae under tension, with subsequent relaxation of the corresponding peripheral lamellar segments. Such an effect is opposite that of the ablation profile to correct hyperopia. Profiles of ablation for hyperopia spare the central corneal zone, but the maximum depth of ablation occurs over a circular line that demarcates the junction between optical and transition zones. The biomechanical changes could have significant consequences, not only on the postoperative apical corneal power, but also on postoperative...

Figure 7. (A) Effect of the variation of the postoperative corneal asphericity on $\Delta C_{20}$ and $\Delta C_{40}$ in an initially oblate cornea. The sign of the variation of $C_{20}^0$ and $C_{40}^0$ depends on the direction of the change of asphericity. When the postoperative asphericity is intended to be more prolate, these variations are negative. (B) Effect of the variation in postoperative corneal asphericity on $\Delta C_{20}^0$ and $\Delta C_{40}^0$ for an initially prolate cornea. The sign of the variation of $C_{20}^0$ and $C_{40}^0$ depends on the direction of the change of the asphericity. When the postoperative asphericity is intended to be more oblate, these variations are positive.

Effect of the variation of the postoperative corneal asphericity on Delta Z20 and Delta Z40 for an initially oblate cornea ($Q=0.2, R=7.8$)

Effect of the variation of the postoperative corneal asphericity on Delta Z20 and Delta Z40 for an initially prolate cornea ($Q=-0.2, R=7.8$)
corneal asphericity. In the case of a customized profile intended to induce an oblate postoperative profile, the additional amount of ablated tissue required to increase postoperative corneal asphericity could have its own biomechanical effect and decrease in turn the predictability of the treatment of the spherical component.

Several studies have emphasized the role of the epithelium in the regression of the refractive effect after excimer laser corneal remodeling. The factors that may be associated with increased epithelial thickness after excimer laser surgery are small ablation zones, greater attempted corrections, and deeper ablations. Conversely, larger, smoother ablation profiles may result in less epithelial hyperplasia. The wound-healing process in PRK and rearrangement of the flap in LASIK may result in partial compensation of increased curvature after laser ablation. Regression after PRK for hyperopia is proportional to the amount of attempted correction. In rabbit corneas, wound healing after PRK for hyperopia has been shown to approximate a lenticular stromal regrowth of approximately 50% of ablated tissue. The healing response of epithelial hyperplasia and stromal remodeling may also contribute to the increased prolateness after LASIK for hyperopia. This is consistent with our previous report showing that the asphericity change in patients treated for hyperopia with LASIK was related to the magnitude of attempted corrections, but not to preoperative or postoperative asphericity. Huang et al. have proposed a mathematical model of corneal surface smoothing after excimer laser refractive surgery. For ablation of hyperopia, the peripheral thickening of the epithelium induced a prolate spherical aberration in their model.

When using paraxial or aspheric eye models to establish theoretical profiles of ablation, the defocus is directly related to the reciprocal of the apical radius of curvature. However, when expressed in the wave aberration theory, the defocus is related to a parabolic function for a rotationally symmetric system. In the presence of spherical aberration, the plane of best image quality may vary with the criteria used to define image quality, aberration level, pupil size, and spectrum of the spatial frequencies forming the image. Depending on the coefficient used to weight the spherical aberration term, Atchison and Smith noted that a change in pupil diameter from 2 to 6 mm would lead to a change in focus of 0.14 to 0.29 D. Their analytical calculation was achieved by using a Taylor expansion to express wave aberration, in which the spherical aberration is only proportional to the fourth power of the ray height in the pupil. It has a different analytical expression than that of the Zernike expansion, which also includes a parabolic term in the expression of the spherical aberration to satisfy the constraint of orthogonality. Thus, similar calculations using Zernike expansions would lead to slightly different results.

The results presented by Atchison and Smith are valid in the pupil plane and do not take into account the presence of nonradially symmetric aberrations. Thus, they cannot be extrapolated for the estimation in the image plane of image-quality metric, such as the point-spread function or the volume under the modulation transfer function. Guirao and Williams have shown that pupil plane methods predict subjective refraction poorly and that the mean absolute error of the prediction of the spherical equivalent increases with increases in higher-order aberrations. Thus, higher-order aberrations influence the amount of sphere and cylinder required to correct vision, and subjective refraction can be better predicted from the eye's optics alone by optimizing computed retinal image quality.

Refraction measured with letter targets is relatively insensitive to pupil size, even in the presence of substantial spher-
ical aberration. This may be due to the high proportion of high spatial frequencies in the small-letter charts. The refraction varies with the number of cycles per degree in some subjects with large pupil diameters, with the optimum focus for lower spatial frequencies being more affected by spherical aberration.

Corneal customized ablations based on wavefront analysis and/or corneal topography represent promising steps toward improving management of hyperopia. A larger optical zone may be important in reducing postoperative spherical aberrations. Before substantial prevention of postoperative aberrations can be achieved, understanding the discrepancy between our preliminary predictions and the clinical observations may be necessary. In addition, further studies addressing the changes in the corneal profile and in the wavefront aberration are needed to confirm these theoretical predictions.

References

Appendix A
A conic section can be described mathematically by Baker’s equation (equation 2), which solves for Z:

\[
Z(x, y) = \frac{-R + \sqrt{(R^2 - (1 + Q)(x^2 + y^2))}}{1 + Q}
\]

where x and y are the coordinates on a Cartesian system with the axis of revolution, R is the apical radius of curvature, and Q is the asphericity. When Q < 0, the ellipse is prolate and...
flattens from the center to the periphery. When \( Q = 0 \), the ellipse is a circle. When \( Q > 0 \), the ellipse is oblate and steepens from the center to the periphery.

When a correction of \( D \) diopters is simulated using equation 1 of Munnerlyn et al. on a cornea modeled as a conic section of apical radius \( R_1 \) and shape factor \( P_1 \), within an optical zone diameter \( S \), the resultant curve \( Z_2 \) is derived from

\[
Z_2(x, y) = Z_1(x, y) + dZ(x, y)
\]

(10)

As shown in a previous analysis of myopia corrections, these equations do not describe a conic section. However, the radius of curvature for each point of the curve \( Z_2 \) is given by \( r_2(x, y) \), which can be computed as

\[
r_2(x, y) = \left( \frac{1 + (Z(x, y))^2}{Z_2^2} \right) \frac{1}{Z_2^2 - (x^2 + y^2)^2}
\]

(12)

After respective first and second derivatives of function \( Z_2(x, y) \) are computed and inserted into the formula \( r_2(x, y) \), the radius of curvature can be calculated by substituting \( Z_2^2 \) and \( Z_2^2 \) in equation 12. The apical radius of curvature of \( Z_2(x, y) \) is \( r_2(0) \). It is calculated by substituting \( 0 \) for \( Z \) and leads to the result

\[
x = 0 \quad \text{and} \quad y = 0, \quad r_2(0) = R_2
\]

(13)

Thus, the radius of curvature of \( Z_2 \) at the apex (apical radius of curvature) is the same as the final radius of curvature, which is derived from equation 1 (Munnerlyn et al.).

Thus, \( Z_2(x, y) \) has an apical radius of curvature \( R_2 \), but its asphericity cannot be computed by the foregoing calculations, because \( Z_2(x, y) \) does not describe a conic section. However, a best-fit conic section, \( C_2(x, y) \), with apical radius of curvature \( R_2 \) and asphericity \( Q_2 \) can be calculated. We plotted multiple conic sections \( C(x, y) \) with asphericity \( Q_2 \). Substituting \( C_2 \) for \( Z \) in equation 9

\[
C_2(x, y) = \frac{R_2 - \sqrt{(R_2^2 - (1 + Q_2)(x^2 + y^2))^2}}{1 + Q_2}
\]

(14)

To determine the best-fit conic section with shape factor \( Q_2 \), we minimized the sum of the squared residuals \( T(Q_2) \).

We developed a numerical procedure and performed computations on a computer spreadsheet (Excel 97; Microsoft, Seattle, WA). We defined 31 values for \((x^2 + y^2)^{1/2}\) ranging from 0 to 6 mm, spaced by 0.1 mm. For a given \( R_1 \), \( D \), and \( Q_1 \), \( T(Q_2) \) was iteratively calculated for \( Q_2 \), ranging from \( Q_1 - 2 \) to \( Q_1 + 2 \) in 0.01 increments. Solutions were represented by the \( Q_i \) that induced the smallest \( T(Q_i) \) (Fig. 2B). \( Q_i \) was tabulated for various \( D \) (+1 to +6 D, by 1-D steps), \( Q_1 \) (−0.6 to 0.4, by 0.2 steps), and \( R_1 \) (6.8–7.8 mm, by 1-mm steps).

We also derived an approximation of \( Q_2 \) in the special case of Munnerlyn-based ablation applied to an aspheric surface using the Taylor series expansion described previously.

\[
dZ \equiv \frac{S_D}{3} + \frac{3S_D^2}{16R_1^2} \left( \frac{S_D}{3} \right) + \frac{3Q_1S_D^2}{16R_1^2} \left( \frac{S_D}{3} \right) + \frac{dQ_1S_D^3}{128R_1^3}
\]

(8)

A spherical ablation is equivalent to an aspheric ablation in the special case where \( Q_1 = 0 \) and \( dQ = 0 \).

The approximation of equation 8 was equalized to that of a spherical ablation \( dZ: Q_1 = Q_2 = 0 \):

\[
dZ \equiv \frac{S_D}{3} + \frac{3S_D^2}{16R_1^2} \left( \frac{S_D}{3} \right)
\]

(15)

Equalizing equation 15 from equation 8 gives

\[
\frac{3Q_1S_D^2}{16R_1^2} \left( \frac{S_D}{3} \right) + \frac{dQ_1S_D^3}{128R_1^3} \equiv 0
\]

(16)

Solving equation 16 for \( dQ \) results in

\[
dQ \equiv -8DR_1Q_1
\]

(3)

**APPENDIX B**

**Zernike Expansion of the Conic Equation**

The Zernike polynomials are defined over the continuous unit disc and normalized. The expansion of any function expressed in polar coordinates \( f(\rho, \theta) \) in a linear combination of Zernike polynomials can be written as

\[
f(\rho, \theta) = \sum_{n=0}^{N} \sum_{k=0}^{N} C_{n}^{\rho-2k} Z_{n}^{\rho-2k}(\rho, \theta)
\]

(17)

where the coefficient \( C_{n}^{\rho-2k} \) is computed as the inner product of \( f \) and \( Z_{n}^{\rho-2k} \).

\[
C_{n}^{\rho-2k} = \langle f, Z_{n}^{\rho-2k} \rangle = \frac{1}{\pi} \int_{0}^{2\pi} \int_{0}^{1} f(\rho, \theta) Z_{n}^{\rho-2k} \rho d\rho d\theta
\]

(18)

In a Cartesian system over a normalized unit pupil the profile of any meridian of a corneal surface with symmetry of rotation centered on the z-axis can be expressed as a conic section, the equation of which is

\[
f(\rho) = \frac{-R + \sqrt{R^2 - (1 + Q)^2\rho^2}}{1 + Q}
\]

with \( 0 < \rho < 1 \) (19a)

The second-order Taylor expansion of the equation of the conic section is

\[
f(\rho) \approx \frac{\rho^2}{2R} + \frac{(1 + Q)}{8R^3} \rho^4
\]

(19b)

where \( R \) and \( Q \) are the apical radius of curvature and the asphericity of the conic section, respectively.

Because of the rotational symmetry of the system, the Zernike expansion of \( f(\rho) \) over a normalized pupil is:
\[ f(p) = \sum_{p=0}^{\lfloor N/2 \rfloor} C_{2p} Z_{2p}(p) \]  

with

\[ Z_{2p}(p) = \sqrt{2p+1} \sum_{l=0}^{p} \frac{(-1)^l (2p-2l)!}{l! (p-l)!^2} \rho^{2l-2l} \]

\[ N \] is the maximum radial order of the Zernike polynomials considered for the expansion.

The coefficients \( C_{2p} \) are computed as

\[ C_{2p} = 2 \int_0^1 f(p) Z_{2p}(p) \rho dp \]

which gives, using the Taylor expansion

\[ C_0 = \frac{1}{4r} + \frac{(Q+1)}{24r^3} + \frac{(Q+1)^2}{64r^5} \]
\[ + \frac{(Q+1)^3}{128r^7} + \frac{7(Q+1)^4}{1536r^9} \]

\[ C_2 = \frac{\sqrt{3}}{12r} + \frac{\sqrt{3}(Q+1)}{48r^3} + \frac{3\sqrt{3}(Q+1)^2}{520r^5} \]
\[ + \frac{\sqrt{3}(Q+1)^3}{192r^7} + \frac{5\sqrt{3}(Q+1)^4}{1536r^9} \]

\[ C_4 = \frac{\sqrt{5}(Q+1)}{240r^3} + \frac{\sqrt{5}(Q+1)^2}{320r^5} \]
\[ + \frac{\sqrt{5}(Q+1)^3}{448r^7} + \frac{5\sqrt{5}(Q+1)^4}{3072r^9} \]

\[ C_6 = \frac{\sqrt{7}(Q+1)^2}{2240r^5} + \frac{\sqrt{7}(Q+1)^3}{1792r^7} + \frac{5\sqrt{7}(Q+1)^4}{9216r^9} \]

\[ C_8 = \frac{(Q+1)^3}{3576r^9} + \frac{(Q+1)^4}{3072r^9} \]

\[ C_{10} = \frac{\sqrt{11}(Q+1)^4}{101376r^9} \]

These coefficients are the coefficients \( C_n \) of the rotationally invariant Zernike expansion terms (\( Z_n \)) of the corneal profile modeled as a conic section. The variations in the corneal wavefront (\( \Delta C_n \)) were calculated for each term by subtracting the postoperative from the preoperative value.