Constructing Retinal Fundus Photomontages
A New Computer-Based Method

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Purpose. To develop computer algorithms for reconstructing 24-bit color, wide-angle composite retinal fundus images from a set of adjacent 45° fundus slides. The authors present the description, technical details, and results of the image reconstruction technique.

Methods. Patients with retinal degeneration underwent fundus photography with a 45° field-of-view fundus camera. Individual photographic slides were digitized for creating fundus montages. Background variations in individual 45° images were modeled to first- or second-order two-dimensional polynomial functions to generate a background image. The background image was subtracted from the original image to obtain background corrected image. Background corrected images were registered and spatially transformed using a first- or second-order two-dimensional polynomial warp model to reconstruct a composite retinal fundus montage.

Results. The authors successfully reconstructed 24-bit color, 100° field-of-view, composite retinal fundus images. The computer-reconstructed montages are an improvement over manually generated montages because computer analysis can be performed on the computer-based montages. In addition, background variations and discontinuities between individual photographs observed in manually generated montages are reduced greatly in computer-generated montages. Most important, the computer-generated montages are better aligned than the manually generated photomontages.

Conclusions. This method of reconstructing a wide-angle composite retinal fundus image from a set of adjacent small- and wide-angle fundus slides is a new tool for creating montages as large as 100° field of view. The computer-generated montages may be used for documenting and quantifying retinal findings. This can greatly assist studies of retinal manifestations of diseases, such as gyrate atrophy, retinitis pigmentosa, sickle cell disease, and acquired immune deficiency syndrome. Invest Ophthalmol Vis Sci. 1996;37:1675–1683.

Fundus imaging has been used to document and monitor retinal changes in persons with diseases such as age-related macular degeneration, diabetic retinopathy, retinitis pigmentosa, acquired immune deficiency syndrome, and gyrate atrophy. With the development of image processing equipment and techniques, researchers and clinicians have begun to develop new ways of imaging the fundus to observe and measure retinal changes in these diseases.

Our interest has been in monitoring changes in the midperipheral and peripheral retina, specifically, the chorioretinal manifestations of gyrate atrophy—a rare, genetically inherited disease. Generally in patients with gyrate atrophy, the chorioretinal dystrophy begins with selective involvement of peripheral retinal pigment epithelium followed by progression of the chorioretinal atrophy toward the posterior pole. Currently, the most widely used method of observing and recording the diseased fundus of patients with gyrate atrophy is fundus photography and photomontages. Fundus photographs of adjacent, overlapping fields
are pasted together to form a retinal photomontage. The clinician then visually monitors the change in the extent of chorioretinal atrophy.1-3

Such manually generated photomontages, however, have certain inherent deficiencies. Color and illumination levels are uneven across the entire montage. Most important, the pictures in the periphery are misaligned. Also, because photomontage attempts to create a single point-of-view picture from photographs with different views, individual pictures in the periphery must be stretched. Because stretching photographs is not possible, the pictures in the periphery are misaligned. This misalignment could potentially lead to underestimation or overestimation of the extent or change in the atrophic regions. Therefore, we explored the usefulness of a computer-based image reconstruction technique to create a mosaic of the fundus (photomontage) from individual fundus pictures.

Fundus imaging has been used widely in the past to document and monitor retinal manifestations of various diseases. Previous works include the use of fundus camera and confocal scanning laser ophthalmoscope to assess optic nerve head changes in patients with open-angle glaucoma4-5; use of fluorescein angiography to examine macular edema, retinal vascular changes in patients with diabetic retinopathy; and use of fundus photography in documenting chorioretinal atrophy in patients with gyrate atrophy.1 Some of these applications4 include image registration techniques used to align images for monitoring retinal pathology. Although some rely on registration during the digitization process,7 others rely on computerized manipulation of images after digitization.8-10

All these applications address the problem of analyzing images of the central retinal region taken at different times. In patients with gyrate atrophy, the area of primary interest is in the midperiphery and periphery. Because a single field of the fundus camera cannot cover the entire fundus, multiple overlapping fields are photographed to recreate the fundus. Therefore, besides a need to register images taken on different visits, there is a need to register images taken on the same visit for montage creation. In this article, we present a description, technical details, and results of a computer technique for creating montages.

**MATERIALS AND METHODS**

Data for mosaicking were obtained from two patients (four eyes) who participated in a clinical protocol approved by the Intramural Review Board of the National Eye Institute. Informed consent was obtained for all patients, and the tenets of the Declaration of Helsinki were strictly followed. The patients’ eyes were fully dilated using three applications of tropicamide 1% and phenylephrine hydrochloride 2.5% solution.

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**FIGURE 1.** Steps in creation of computer mosaic of fundus photographs.

Each patient’s fundus was photographed onto Kodachrome 64 color slide film (Eastman Kodak, Rochester, NY) using a 45° field-of-view (FOV) fundus camera, the Nikon Retinapan 45-II (Nikon, Torrance, CA). Color slides were digitized as 24-bit color images using a Nikon LS-3510AF slide scanner (Nikon) and Adobe PhotoShop software version 2.5 (Adobe Systems, Mountain View, CA). These scanned images served as the data for reconstruction.

Scanned images were processed using the newly developed National Eye Institute Retinal Fundus Imaging software. This software, an extension to the NIH Image version 1.60 software package (Wayne Rasband, NIH, ftp: zippy.nimh.nih.gov), includes background correction and image reconstruction routines developed on a PowerMac 8500/100 computer (Apple, Cupertino, CA).

The image reconstruction method, referred to as mosaicking, uses a two-dimensional polynomial warp model for creating a mosaic from individual images. Mosaicking includes the following major steps: fundus photography, slide digitization, background correction, and image registration. A flow chart of these steps is shown in Figure 1. The section below describes step 4, spatial image registration and mapping. The
second section describes step 3, techniques used to reduce background variations.

Spatial Image Registration and Mapping

Fundus imaging is a case of representing a spherical surface—the retina—onto a flat plane. Any representation of the sphere on a flat surface is bound to have distortions. An analogous example is the problem experienced by cartographers in representing earth’s surface on maps. At best, based on the application and region of the sphere, a projection system may be used that minimizes the distortion in the particular region. The common projections used by cartographers to represent earth are variations of azimuthal, cylindrical, and conical projections, each of which introduces systematic distortions that may be calculated mathematically.11

Because it is difficult to photograph the entire fundus as a single photograph, a series of photographs of adjacent areas is acquired by altering relative positions of the camera and the subject. The result is a set of photographs that approximate oblique azimuthal projections of the fundus. One method to create a composite image of the fundus would be to recreate the three-dimensional image of the fundus from these individual oblique projections. The three-dimensional image could be used to create projections that minimize distortion, or it could be used directly to visualize and measure retinal changes.

Another method is to use one of the oblique projections as the anchor image and to modify other pictures so that their viewing geometry approximates that of the anchor image. The process of creating a photomontage and the equivalent computer method—image mosaicking, described below—uses this latter approach. In such an approach, the distortion is smallest in the center of the anchor image and increases as one moves away from the center. When the picture centered on the orbital pole of the eye is chosen as the anchor image, the resultant mosaic would be a close approximation of the polar azimuthal projection of the fundus. Choosing any other anchor image would lead to an oblique azimuthal projection. The montages depicted in this article fall in the former category—approximate polar azimuthal projections.

Image mosaicking involves techniques to “splice” or to register geometrically an ensemble of narrow-angle images to form a wide-angle composite or mosaic. Image mosaicking includes geometric view correction and spatial image registration. Geometric view correction is a spatial image transformation step that ensures individual images have a common viewing geometry. The second step, spatial registration, identifies the spatial relationship between a pair of modified images. One method to correct view geometry of images is to build a mathematical model of fundus imaging. The mathematical model can then be used to correct or change the viewing geometry of individual images to a common viewing geometry. Once the view geometry of the image has been corrected, the two images can be registered spatially and merged.

Fundus imaging is a case of azimuthal projection of a spherical object, the human retina, onto a two-dimensional plane, and it can be expressed by a general form of geometric image formation model described by equation 1.

\[
x_i = g(x_o, \ldots)
\]

where \(x_o = \begin{bmatrix} x_o & y_o & z_o \end{bmatrix}\) and \(x_i = \begin{bmatrix} 0 & y_i & z_i \end{bmatrix}\) are the three-dimensional and two-dimensional geometric coordinates of the scene and its image, and \(g\) is a “noninvertible” geometric transformation that relates the points, \(x_o\), in the three-dimensional scene to image points, \(x_i\), on the two-dimensional image plane. The transformation function \(g\) is dependent on the imaging geometry and on the magnification of the imaging system, which includes the camera and human lens.12 The transformation is noninvertible because it is not possible to recreate a three-dimensional image from the two-dimensional fundus picture. However, a knowledge of the model \(g\) and the parameters of the model will allow geometric view correction of images.

Although researchers have developed models for the magnification of the human lens and camera imaging system,\(^{13,14}\) a complete fundus image formation model is unavailable. The major constraint to a universal image formation model is the difficulty in recording in vivo the relative geometry of the fundus and the camera. A lack of reference points to ascertain the relative position of the camera and the human fundus, as well as eye movement, are the key factors contributing to the lack of a universal image formation model.

Under these circumstances, it is desirable to have a method for geometric view correction that does not require a priori knowledge of the geometric transformation parameters. Such a method should rely on information intrinsic to the image. One method is to model the image transformation with a general mathematical model whose parameters are identified by using data intrinsic or extrinsic to the images. Some of the more commonly used mathematical models for image transformation are the affine transform, perspective transform, and the polynomial transform.15

In all these models, the data from images are analyzed to register images and to identify the required image transformation. The image registration algorithms may use either part of or the entire image data leading to global and local image registration meth-
Global registration methods, such as cross-correlation-based registration, rely on all the pixels in the image for registration and are, hence, computationally expensive. On the other hand, local methods use a subset of the image data for registration. Here, distinctive features, such as ocular blood vessels and/or their crossings or atrophic regions, are extracted. Image registration is performed only on these local features. The method described here, the polynomial warp model, uses the local approach for image registration. Polynomial warping has been used extensively in satellite imaging and simulation applications to create a wide FOV image mosaic from narrow FOV images. In this article, we describe a new application in the field of ophthalmology.

**Polynomial Warp Model-Based Image Mosaicking.** Polynomial warp model-based image mosaicking starts with an assumption that two images that are spliced have an overlap. Data from this common area is used to determine the required image transformation. On transformation, the new or correctly registered image has the same viewing geometry as the first image and, therefore, can be merged into the first image—the master image.

If functions \( f(x) \) and \( f(w) \) represent the two images, where image 1 is recorded in \((x_1, x_2)\) and image 2 is recorded in \((w_1, w_2)\) coordinate system, the spatial relationship between the independent variables of the two images can be expressed by the general polynomial model described in equations 2 and 3 (see Appendix A for a more detailed description):

\[
\begin{align*}
    w_1 &= \sum_{i=0}^{N} \sum_{j=0}^{N} k_{ij} x_i^ix_j^j \quad (2) \\
    w_2 &= \sum_{i=0}^{N} \sum_{j=0}^{N} k_{ij} x_i^ix_j^j \quad (3)
\end{align*}
\]

A set of \( N \) control point pairs \((x_{1n}, x_{2n})\) and \((w_{1n}, w_{2n})\), representing common features in the two images, are identified. Coordinate data from the control point pairs are substituted in equations 2 and 3 to obtain \( N \) linear equations of parameters \( k_{ij} \). Solution of these linear equations using the singular value decomposition method (SVD) identifies parameters \( k_{ij} \). Although control point identification could be automated, the system described here uses an interactive approach that relies on manual control point identification.

The SVD method for solving linear equations attempts to "fit" the given data to a mathematical model specified by the user. Besides evaluating the parameters of the equation, the SVD method evaluates a "figure of merit function" that measures agreement between actual data and model-based data. The figure of merit function—in this instance, "least squares error"—serves as the decision variable for selecting the "best fit function," among a set of functions.

Although equations 2 and 3 can be expanded into innumerable possible polynomial models, the highest degree of a usable polynomial model is influenced by the number of unique control points in two adjacent images. The higher the degree of polynomial, the greater the number of coefficients that must be evaluated and, hence, the greater the number of required control point sets. In addition, higher order polynomials induce their own distortions during image transformation. In most instances, a second-degree polynomial model is sufficient. The system described here examines a set of five first-and second-degree polynomial functions to determine the "best" transformation model (see Appendix B for a listing of functions used), where the best model is the one with smallest least squares error.

**Background Correction**

Most image analysis procedures use brightness and color levels of regions in an image to identify features of interest. The analysis assumes that a feature will have the same brightness and color levels wherever it appears in the FOV of the camera. Under ideal conditions, when the fundus is evenly illuminated, this may be true; however, in practice this rarely happens. Significant background variations almost always exist within images. A technique is needed then to correct background variations within images. In addition to variations within an image, differences can exist between the brightness and color levels of adjacent images. When merging two images, these differences must be minimized. This section describes the techniques used for correcting color variations that arise because of the above-described conditions.

**Background Correction Within Image.** Various techniques exist for correcting background variations within an image. Spatial filtering techniques may be used to remove small regional variations or noise. Frequency domain filtering techniques based on fast Fourier transform may be used to remove background variation when either the background or the subject is repetitive. A rolling ball technique can remove background variation when the subject of interest is small relative to the size of the image.24 In some instances, the background can be imaged independently and subtracted from the image to get a background corrected image.25 Another technique to correct background variations entails developing a mathematical model for background variations. The model is then used to create a background image, which is subtracted from the original image to obtain a background corrected image.25 In the mathematical model-based technique, a series of regions representative of the image background is identified, and the
FIGURE 2. (A) Image with severe background variations; (B) background corrected image.

FIGURE 3. (A,B) Two adjacent images for patient 1 showing manually registered control points.

background levels in these regions are used to identify appropriate model.

Because background variations observed in fundus images were primarily caused by uneven illumination, the model-based technique was found to be most appropriate. The mathematical model used is a general form of polynomial equation described by equation 4. The SVD technique, used for image mapping and registration, and the five polynomials shown in Appendix B, are used for background correction.

\[
\text{Background}(w_1, w_2) = \sum_{i=0}^{N} \sum_{j=0}^{N} k_{ij} w_1^i w_2^j
\]

Background Correction Between Images. In addition to variations within images, there can be significant brightness and color differences between adjacent images. When merging adjacent images, the color levels of the images must be adjusted to minimize variations in the merged image. The National Eye Institute system relies on the mean color statistic of overlapping areas in two images to modify pixel color. In addition, a weighted linear ramp function is used to weigh proportionally pixels from both images in a 10-pixel-wide band along the edges where the two images join. This ensures that discontinuities along the edge are less visible.

RESULTS

Using the technique described in the previous sections, computer fundus mosaics for six patient eyes
were completed. The intermediate and final results for two patient eyes are described below. The first step of the mosaicking process was removal of background variations within the image. Figure 2A shows a raw 45° photograph with significant background variation observed at the bottom right corner of the image. An operator (AAM) identified 12 background points by placing the cursor over the region and marking the control point. A 3 x 3 pixel window centered on the control point was used to compute the mean color levels at that point.

Data from these control points were used to identify the parameters of the five polynomial models. The best model was used to generate a background image. Background corrected image (see Fig. 2B) was obtained by subtracting the background image from the original image. As seen in the two figures, background subtraction leads to an image whose color is uniform, and features obscured by background variations become visible in the corrected image.

The second step of the mosaicking process was spatial mapping and registration. For this, a series of landmark points representing common features in two adjacent images were identified manually by the operator (AAM). Typical landmark points were blood vessel crossings or other discernible features, such as atrophic lesions. Figures 3A and 3B show the nine manually registered control points for two adjacent images. Data from the nine control points were used to identify the parameters of the best image transfo-
Retinal Fundus Photomontages

The image transformation model was used to transform pixels of registration image, its color altered by using differences in mean color levels, and was merged into the master image. The software allows the following strategies for processing pixels in the common areas of the two images: Retain pixels of the anchor image; replace pixels of anchor image with peripheral image pixels; and replace anchor image pixels with an average of anchor and peripheral image pixels. In addition to these options, the user can delete selectively portions from anchor or peripheral pictures to replace or retain specific areas from the two images. This feature is particularly useful when the edges of both the anchor and the peripheral images are dark, as happens often in peripheral fundus photographs. The montage examples shown here use this selective deletion approach. Areas from the anchor image that were unclear were erased and replaced with pixels from the peripheral image. In all other common areas, the pixels of the anchor image were retained.

This procedure was repeated on all the raw images of patient 1. The resultant computer-generated mosaic is shown in Figure 4B, and the corresponding manually generated photomontage is shown in Figure 4A. The time required to create computer montages is comparable to manual montage creation; it takes less than 1 hour on the PowerMac and can be further reduced with greater automation.

As seen in Figure 4, the computer-generated mosaic is a significant improvement over the manually generated montage. Color variations across the image are minimized, the edges smoother, and, most important, the images are well aligned. An illustration of this is the discontinuity observed in the blood vessels above the optic nerve in the manually generated montage shown in Figure 4A. The same blood vessel is aligned in the computer montage shown in Figure 4B.

Although the system selects a default image transformation model, it also provides the user with an override option. User override is useful in instances in which overlap between adjacent images is minimal, leading to a situation in which the computer model may be less accurate. This problem can be overcome by having multiple pictures taken with sufficient overlap. Another instance in which manual override is useful is the registration of images in the far periphery. Because azimuthal projection of human fundus introduces distortions as one moves away from image center, the mosaic becomes increasingly distorted in the far periphery (beyond 100° FOV). The peripheral distortion in the computer-generated mosaic can be seen in Figure 4B, in which peripheral lesions appear compressed in the periphery. In such instances, the operator may override the computer-generated model to minimize distortions. However, this again leads to the problem of misaligned images. The problem of peripheral distortion is difficult to overcome beyond a certain extent, limiting the FOV of the present system to approximately 100°.

One of the effects of background correction is the modification of colors in the corrected images. This is observable when one compares the two images in Figures 4A and 4B. For studying gyrate atrophy lesions, this is not a major concern, but in studying microaneurysms or soft exudates observed in patients with AIDS, the color shift may be of concern. In such instances, the software may be instructed to skip the background correction step, thereby retaining "original" colors.

Figures 5A and 5B are the manual and computer-generated montages, respectively, for patient 2. The computer-generated montage shown in Figure 5B is truncated at the 100° FOV. To measure system reproducibility, the fundus of patient 2 was photographed within 24 hours, and the montage was constructed. The two montages were then registered and aligned to reflect the same viewing geometry. The nonatrophic region surrounding the optic nerve in the registered montages was traced manually, and the area was measured in pixels. The same procedure was repeated on the other eye of patient 2. The area measures for the two eyes were compared and found to be within 5% of each other.

**DISCUSSION**

The wide, ≈100°, field-of-view computer-generated montages are a significant improvement over manually generated photomontages. Compared to manual montages, discontinuities between images are less pronounced, colors are more uniform, and images are aligned better. The montages permit better visualization and quantification of atrophic regions in the near peripheral fundus of patients with gyrate atrophy. One method of visualizing changes in retinal pathology is to superimpose montages from two visits and perform arithmetic subtraction of the two images. The subtracted image would highlight the changes in the lesions.

Quantification could be area measurement of nonatrophic region around the optic nerve, distance measurement of a particular advancing lesion from the optic nerve, or area measurement of atrophic lesions. One of the problems of fundus photography is the difficulty in ensuring that the extent of the field photographed on two visits is the same. This makes
area measurement of the lesions in the periphery difficult. Area measurements of lesions entirely visible in the field are, however, possible. Therefore, measurement of nonatrophic regions around the fovea may be the most attractive option. For examining system reproducibility, the first approach—measurement of nonatrophic region around fovea—was used.

Although this system was developed for creating fundus photomontages of patients with gyrate atrophy, it is not limited to gyrate atrophy. The method can be used in any instance in which a large FOV image is reconstructed from smaller FOV images. Some instances in which computer-generated montages may be used for disease documentation are observing retinal changes caused by cytomegalovirus retinitis in patients with AIDS; observing retinal changes caused by diabetic retinopathy (similar to the attempts of Early Treatment Diabetic Retinopathy Study, in which seven fields of the fundus are photographed to document retinal changes); and retinitis pigmentosa.

Presently, most fundus photography is accomplished using film cameras. The use of digital cameras—gray scale and color—in fundus photography, however, is increasing rapidly. Digital imaging can be combined easily with the above system to provide a cost-effective method for documenting retinal manifestations of various diseases. One area of improvement in the present system may be greater automation. For instance, the system described here uses an interactive approach for control point identification. Image registration methods, such as auto-correlation, will permit greater automation.

The polynomial warp model mosaicking produces an azimuthal projection of the human fundus that introduces distortions as one moves away from the center of the image. For instance, a north pole azimuthal projection of earth’s surface introduces distortions as we move away from the pole. This property of azimuthal projections limits the FOV of the present system to 100°, beyond which the montage is increasingly distorted. In situations in which the feature of interest lies beyond this FOV, it is possible to create an oblique azimuthal projection centered on the particular lesion. This will minimize distortion in the particular area of interest.

Another method to minimize distortion is to use a different projection, such as conic or spherical, or a special azimuthal projection, such as the equal area projection. Montages from these projections are much more difficult to create. An alternative approach may be a three-dimensional reconstruction of the retina, which would eliminate any distortions caused by two-dimensional methods. Storage space requirements, lack of depth information in fundus photographs, and inability to record the relative position of camera and eye, make three-dimensional reconstruction difficult.

Until these issues are resolved, the mosaicking method described may be one solution for generating computer-based montages. The relatively low costs of desktop computers and slide scanners used in our setup permit even small laboratories to use such a system for clinical studies. Further, many new fundus imaging systems include computers that may be used to run this software, lowering costs even further. The computer-generated montages could then be used for monitoring retinal manifestations of diseases such as gyrate atrophy, AIDS, diabetic retinopathy, retinitis pigmentosa, and sickle cell disease.

**Key Words**

fundus photograph montages, fundus photography, gyrate atrophy, retinal image analysis, retinitis pigmentosa

**References**


†This software, which runs on either a 68000-series or PowerPC-based Macintosh, is in the public domain and may be obtained from the authors.


**APPENDIX A**

The general equation relating the two individual images spatially is expressed by equation 5.

\[ x_i = g_i(w_1, w_2) \]  

(5)

The function \( g \) in the case of a polynomial warp model is expressed by equations 6 and 7.

\[ x_1 = \sum_{i=0}^{N} \sum_{j=0}^{N} k_{ij} w_1^i w_2^j \]  

(6)

\[ x_2 = \sum_{i=0}^{N} \sum_{j=0}^{N} k_{ij}^2 w_1^i w_2^j \]  

(7)

Given a set of \( N \) control point sets in the \( x \)- and \( w \)-based coordinate systems of images 1 and 2, respectively, and for a degree of \( N = 2 \), the general equation can be expanded into the expression shown in equations 8 and 9.

\[ x_{1k} = k_{00} + k_{01} w_{1k} + k_{02} w_{2k} + k_{11} w_{1k} w_{2k} + k_{00} w_{1k}^2 + k_{02} w_{2k}^2 + k_{11} w_{1k} w_{2k} 
+ k_{12} w_{1k}^2 w_{2k} + k_{22} w_{1k}^2 w_{2k}^2 \]  

(8)

\[ x_{2k} = k_{00} + k_{10} w_{1k} + k_{02} w_{2k} + k_{12} w_{1k}^2 + k_{00} w_{1k}^2 + k_{02} w_{2k}^2 + k_{12} w_{1k}^2 w_{2k} 
+ k_{12} w_{1k} w_{2k}^2 + k_{22} w_{1k}^2 w_{2k}^2 \]  

(9)

A substitution of the data from \( N \) control points leads to \( N \) linear equations of the parameters \( k \). These linear equations can be solved to identify the values of the parameters \( k \) and then the model for image mapping.

**APPENDIX B**

From the general polynomial model described in equations 8 and 9, the entire function or subsets of terms may be used to model the spatial relationship between two images. Table 1 shows the five functions used in our method.

<table>
<thead>
<tr>
<th>Number</th>
<th>Function</th>
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<tbody>
<tr>
<td>1</td>
<td>( f(x, y) = a_1 + a_2 x + a_3 y )</td>
</tr>
<tr>
<td>2</td>
<td>( f(x, y) = a_1 + a_2 x^2 + a_3 y^2 )</td>
</tr>
<tr>
<td>3</td>
<td>( f(x, y) = a_1 + a_2 x + a_3 x^2 + a_4 y + a_5 y^2 )</td>
</tr>
<tr>
<td>4</td>
<td>( f(x, y) = a_1 + a_2 x + a_3 x^2 + a_4 y + a_5 y^2 + a_6 x y + a_7 x y^2 + a_8 y^2 )</td>
</tr>
<tr>
<td>5</td>
<td>( f(x, y) = a_1 + a_2 x + a_3 x^2 + a_4 y + a_5 y^2 + a_6 x y + a_7 x y^2 + a_8 y^2 )</td>
</tr>
</tbody>
</table>

**TABLE 1. Five Polynomial Equations Used for Image Transformation**