Detection Efficiency of Circular Gratings and Bandpass Filtered Points With Randomized Phase Spectra

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Purpose. In the studies of spatial integration an increase in the spatial extent of the stimulus usually results in a decrease in the spatial frequency bandwidth of the stimulus. The authors investigated separately the effects of these two factors on contrast detectability.

Methods. Efficiencies were measured for a circular grating at 4 c/deg and for bandpass-filtered point stimuli having a constant center frequency at 4 c/deg and bandwidths of 0.25, 0.5, 1, and 2 octaves. The phase range of these two-dimensional stimuli was increased from zero to 90, 180, 270, and 360 degrees by replacing the original zero phase at each spatial frequency component by a random number with zero mean. This procedure left the spatial frequency bandwidth unaffected.

Results. The increase in phase range and decrease in spatial frequency bandwidth caused a progressively larger proportion of the contrast energy of the point stimuli to spread into their surroundings. As a result, detection efficiency decreased with increasing bandwidth and phase range for all point stimuli. However, a change in the stimulus bandwidth affected efficiency only when it altered stimulus area. The area of the circular grating and its detection efficiency remained almost constant irrespective of the phase range. When efficiency was plotted in semi-logarithmic coordinates as a function of stimulus area expressed in terms of the spatial spread of contrast energy, the line of least squares explained 85% of the total variance.

Conclusion. The primary determinant of detection efficiency for stimuli with constant center spatial frequency is not stimulus bandwidth but stimulus area expressed in terms of the spatial spread of contrast energy. Invest Ophthalmol Vis Sci. 1994; 35:3111-3118.

Spatial integration in the visual system refers to the collection of contrast information over space so that when stimulus area increases, contrast sensitivity increases. The limited extent of spatial integration in the human visual system has been demonstrated by several studies of contrast sensitivity1-4 and detection efficiency.5-7 These studies suggest that for sinusoidal gratings, the limiting factor is not the stimulus area in degrees of visual angle but the number of square cycles5,4 calculated by multiplying grating area by spatial frequency squared. This means that spatial integration in the human visual system is scale invariant.7

An increase in the grating area also reduces the bandwidth of the amplitude spectrum of the stimulus in Fourier space. However, we can increase the stimulus area without affecting the amplitude spectrum by modifying the phase spectrum of a two-dimensional, spatially limited image, such as a bandpass-filtered point stimulus. Each spatial frequency component of an image has its specific phase in Fourier space. Thus, the phase spectrum of the image defines the spatial relationship between its frequency components. When the phase spectrum of a bandpass-filtered point stimulus is modified, for example, by adding a random number to the phase of each spatial frequency component, the stimulus area increases but the bandwidth remains unchanged.

We studied spatial integration in noise for bandpass-filtered point stimuli whose spatial frequency

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bandwidths were kept constant while the areas were increased by modifying the phase spectra. In this way, the effect of stimulus area on detection efficiency could be investigated without a change in bandwidth. In addition, we used a circular grating with a radial luminance modulation as a control stimulus because the randomization of its phase spectrum did not affect the area.

METHODS

Apparatus

Stimuli were generated under computer control on a high-resolution color monitor driven at 60 Hz by a VGA graphics board. The pixel size was 0.42 × 0.42 mm². To obtain a monochrome signal of 1024 intensity levels from a monochrome palette of 65,536 intensity levels, we used a video summation device. The display was used in a white mode. The luminance response was linearized using the inverse function of the nonlinear luminance response when computing the stimulus images. The contrast of simple cosine gratings was independent of spatial frequency and orientation up to 2 c/cm.

The stimuli were drawn on the screen with coordinates (x, y) varying between (0, 0) and (639, 479) by means of software developed by one of the authors (RN). The horizontal and vertical dimensions of the screen were 30 and 21.5 cm, respectively. The average photopic luminance of the display was 50 cd/m² (for a more detailed description of the apparatus, see ref. 9).

Stimuli

Bandpass-filtered points of different bandwidths and a circular grating, all of which had their spatial frequency components in zero phase, were used as basic stimuli for our experiments.

The bandpass-filtered points were obtained from an impulse stimulus with a flat amplitude spectrum. In practice, the local contrast (see equation 3) of only one bright pixel in the middle of the image differed from zero. The discrete Fourier transform of the impulse stimulus was filtered by means of a circularly symmetric log-Gaussian transfer function:

\[ \text{MTF}(f) = e^{-\frac{1}{2} \ln^2 \left( \frac{f_c^2 + f_r^2}{b^2} \right)} \]  

where \( f \) is radial spatial frequency \( [f = (f_r^2 + f_c^2)^{1/2}] \), \( f_c \) is radial center frequency, and \( b \) is half of the spectral bandwidth at half-height in octaves. In our experiments, the radial center frequency was always 1.5 c/cm resulting in 4 c/deg from a viewing distance 154 cm used in the experiments. Bandwidths at half-height of the filter were 0.25, 0.5, 1, and 2 octaves.

From each bandpass-filtered impulse stimulus, four new stimuli were produced by adding an evenly distributed random number with zero mean and range of 90, 180, 270, or 360 degrees to the zero phase value of each spatial frequency component \( (f_r, f_c) \). After discrete Fourier transform, a similar randomization of the phase spectrum was carried out for the circular cosine grating that had a radial spatial frequency of 4 c/deg.

The inverse Fourier transforms then produced the stimuli used in our experiments. As examples in Figure 1 show, the increase in the phase range increased stimulus area for point stimuli and degraded the structure of all stimuli. The grain of image at the phase range of 360° was finer the wider the stimulus bandwidth.

Stimuli were embedded in white, two-dimensional static noise. Noise was produced by adding a normally distributed random number with zero mean to each pixel. The values of the random numbers in the neighboring pixels were uncorrelated. Thus, the noise was white within the frequency range of our stimuli.

Each experimental trial consisted of three rectangular stimulus windows shown simultaneously side by side. The size of each window was 5.4 × 5.4 cm² (128 × 128 pixels), and the inter-center distance was 7 cm. The stimulus windows were surrounded by an equiluminous homogenous field.

The window in the middle of the screen contained a copy of the signal without noise. This model signal was used to minimize the observer’s uncertainty of the stimulus to be detected. On both sides of the model, there was a window containing noise. One of the noise windows also contained the signal to be detected. The contrast of the model signal was always twice the contrast of the signal to noise. This ensured that the model signal was visible even at the threshold of the signal to be detected.

The stimuli were computed and stored on a hard disk. There were 10 stimuli with different noise samples at each contrast level. In five of them, the signal was embedded in the noise window on the left side and in the other five on the right side. For each trial, one of the ten stimuli was chosen at random for presentation. A different set of 10 noise sample pairs (two noise windows in each stimulus) had been generated for each contrast level. In the experiments, about 15 contrast levels were used for measuring a single threshold. Thus, the total number of different noise samples available was about 300 per threshold.

In the experiments, we measured contrast energy thresholds. Contrast energy \( E \) was calculated by integrating numerically the square of the local contrast \( c(x, y) \) of signal across the stimulus area:

\[ E = \sum \sum c^2(x, y) p^2, \]
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FIGURE 1. Examples of the stimuli used. The bandwidth of the stimulus is 2 octaves on the left and 0.5 octaves in the middle. The circular grating is on the right. The phase range increases from 0° (top), to 180° (middle), to 360° (bottom).

where $p^2$ is the pixel area. The local contrast $c(x,y)$ is defined as:

$$c(x,y) = \frac{L(x,y) - L_0}{L_0},$$

(3)

where $L(x,y)$ and $L_0$ are the local and mean luminance, respectively.

The spectral density of white external spatial noise is the product of noise rms contrast ($c_n$) squared and the pixel area ($p^2$): $N_e = c_n^2 p^2$, where the rms contrast of noise ($c_n$) is the standard deviation of the luminance distribution of noise calculated pixel by pixel across the noise window and divided by the average luminance. In our experiments, the rms-contrast of noise was 0.3. The pixel area was $0.0156 \times 0.0156$ deg$^2$ from the viewing distance of 154 cm used in our experiments. Thus, the noise spectral density ($N_e$) was $2.19 \times 10^{-5}$ deg$^2$, which was great enough to make the human contrast energy threshold proportional to the...
spectral density of external noise. Thus, the effect of external noise was large compared with the effect of internal noise.

Procedure
Contrast thresholds for the stimuli were determined using a two-alternative, forced-choice algorithm. The observer had an unlimited viewing time to indicate in which noise window the stimulus was by pressing one of two keys on a computer keyboard. The response terminated the presentation of the stimulus and started a new trial.

In each threshold estimation, the first contrast shown was always clearly above threshold. A threshold estimate at the probability level of 0.84 of correct was obtained as the mean of the last eight turning points. The procedure of threshold estimation has been described in detail elsewhere.

Between trials, the observer saw a homogenous field having the mean luminance of the stimuli. An auditory feedback was given to indicate incorrect responses. The stimuli were viewed binocularly with natural pupils (diameter 5 to 6 mm). Each data point is a geometric mean of six threshold estimates.

The results are expressed in terms of detection efficiency. The efficiency (η) of a real detector (e.g., human) is given by the ratio of the energy thresholds of the ideal and real detectors. Energy threshold for an ideal detector without internal noise is the product of the detectability index (d') squared and the spectral density of external noise (N_e). In our study, the threshold estimates correspond to the probability level of 0.84 of correct responses. From Elliot’s forced-choice tables, the value for d' is 1.4. Hence, efficiency in our study was calculated by

\[ \eta = \frac{2N_e}{E_{\text{human}}} = 2N_eS_E, \]  

where \( E_{\text{human}} \) was contrast energy threshold determined experimentally. Its inverse \( S_E \) is contrast energy sensitivity.

Subjects
Two experienced subjects, who were also authors of the article (HK and RN) and thus were fully aware of the nature of the experimental procedures, served as observers. All procedures followed the principles outlined in the Declaration of Helsinki. Both subjects had normal binocular, refractive, and ocular health status. RN was a corrected myope (−4.25 DS OA) with a binocular visual acuity of 2.0. HK was an uncorrected hyperope (+0.5 DS OA) with a binocular visual acuity of 1.3. The visual acuities were measured with an Oriola test chart based on Sloan letters.

Explained Variances
The goodness of the least-squares curves fitted to the efficiency data was estimated by calculating the variance of the experimental data from the least-squares values and expressing this as a percentage of the total variance of the experimental data:

\[ r^2 = 100 \left[ 1 - \frac{\Sigma ( \log \eta - \log \eta_{\text{est}} )^2}{\Sigma ( \log \eta - \eta_{\text{ave}} )^2} \right] \]  

where \( \eta_{\text{ave}} = n^{-1} \Sigma \log \eta \) and \( \eta_{\text{est}} \) are the least-squares values. We used \( \log_{10} \) instead of \( \eta \), because \( \eta \) is plotted on a logarithmic scale.

RESULTS
Figure 2 shows contrast energy thresholds as a function of the range of phase randomization for circular cosine gratings with a radial luminance modulation of 4 c/deg and for the point stimuli filtered using log-Gaussian transfer functions of various bandwidths and 4 c/deg center frequency. The right vertical axis expresses the results also in terms of contrast energy sensitivity (S_E), which is the reciprocal of energy threshold.

As Figure 2 indicates, the results of the two observers were almost identical. Efficiency and energy sensitivity decreased with increasing phase range for all point stimuli. However, the effect of increasing phase range on efficiency became smaller with decreasing stimulus bandwidth. Comparison with Figure 1 reveals that, in fact, the effect of phase range on efficiency seemed to depend on the change in the stimulus area: For the stimulus with 2 octaves of bandwidth, efficiency decreased from approximately 50% to 1.5%, whereas for the stimulus with narrow, 0.25-octave bandwidth, efficiency decreased from about 20% to 4%. In accordance, the area to which most of the contrast energy was confined increased more with increasing phase range for stimuli with large bandwidths (1 to 2 octaves) than for stimuli with narrow bandwidths (0.25 to 0.5 octaves). Between 270° and 360° of phase range, the decrease in efficiency slowed down and even reversed with decreasing stimulus bandwidth.

The area of the circular grating did not increase with the phase randomization; only the stimulus structure was changed (see Fig. 1). As a result, the increase of the phase range had only a small effect on detection efficiency for the circular grating. Efficiency decreased from 8% to 4.5% with increasing phase range.

As a measure of area to which most of the contrast energy is confined, we have previously used the smallest area that comprises 95% of the total contrast energy of the stimulus (A_95). A_95 is obtained by calculat-
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Figure 2. Detection efficiencies plotted as a function of phase range for point stimuli with various bandwidths and for the circular grating. Contrast energy sensitivity, equal to efficiency multiplied by $22.7 \times 10^5$, is shown on the right vertical axis. The bandwidths of the stimuli and subjects are indicated.

Figure 3A and 3B demonstrate that log detection efficiency for point stimuli decreased rather linearly with increasing $A_{75}$ for both subjects. The efficiency data collected in this experiment had the squared inter-subject correlation coefficient of 92% in double logarithmic coordinates, in agreement with a previous study. The percentage indicates the accuracy by which the experimental data of one subject can be predicted by another and, hence, the maximum explained variance that a line of least squares fitted to the data of two subjects can yield. The pure error in our data is thus of the order of 8% in agreement with the earlier study.

As Figure 3 shows, efficiency decreased approximately linearly in semi-logarithmic coordinates as a function of $A_{75}$ comprising 75% of the total contrast energy of the stimulus. The line of least squares of the form $\eta = \eta_0 \times e^{k A_{75}}$ fitted to the efficiency data of subjects HK and RN explained 66% of the total variance. Thus, the variance that remained unexplained after subtraction of the pure error was 26%. The values of constants $\eta_0$ and $k$ were 0.37 and $-1.6$.

The scrutiny of Figure 3 shows that changes in the bandwidth of the point stimuli did not affect efficiency when $A_{75}$ remained constant. The efficiency data for...
point stimuli with bandwidths ranging from 0.25 to 2 octaves followed the same decreasing line of least squares irrespective of the bandwidth and only depending on \( A_{75} \).

A significant deviation from the linear decrease for the point stimuli was at 2 octaves bandwidth with 360° of phase randomization. In addition, detection efficiency for the circular grating was found to increase with \( A_{75} \), which is the main reason for the poorly explained variances found above. The deviation of circular grating data from the point stimulus data was due to the fact that for the circular grating, \( A_{75} \) decreased, but for point stimuli, it increased with increasing range of phase randomization.

Area measures based on a constant percentage of the total contrast energy of the stimulus have other problems, too. For example, they cannot explain the decrease in detection efficiency with increasing distance between grating patches.\(^{16}\) In such cases, \( A_{75} \) indicates the sum of the areas that each grating patch occupies, and it would not change with increasing distance between the patches. Therefore, we have introduced another measure of area, called the spatial spread of contrast energy (\( \alpha \)).\(^{16}\) It is calculated as

\[
\alpha = \frac{\pi \int \int r(x,y)c^2(x,y)dxdy}{\int \int c^2(x,y)dxdy},
\]

where \( r(x,y) \) refers to the distance from the center of gravity computed on the basis of contrast waveform \( c(x,y) \) of the stimulus. The spatial spread of contrast energy (\( \alpha \)) within the stimulus area is a kind of variance of image distances weighted by local contrast squared and normalized by the total contrast energy of the image.

When we plotted the detection efficiencies of Figure 2 as a function of \( \alpha \) in Figure 4, efficiency again decreased approximately linearly in semi-logarithmic coordinates as a function of stimulus area now expressed in terms of the spatial spread of contrast energy. In this case, the data for circular gratings and point stimuli did not differ significantly from each other. The data point corresponding to point stimulus with two octaves bandwidth and 360° phase range was now the only significant deviation from the linear decrease. The least squares line of the form \( \eta = \eta_0 \times e^{k\alpha} \) fitted to the efficiency data of subjects HK and RN now explained 85% of the total variance, resulting in an unexplained variance of only 7% after subtraction of pure error. The values of constants \( \eta_0 \) and \( k \) were 0.44 and -1.1.

**DISCUSSION**

Detection efficiencies for the bandpass-filtered point stimuli with various bandwidths and for the circular grating with a radial luminance modulation were found to decrease with increasing range of phase randomization. For the point stimuli, an increase in phase range increased stimulus area and reduced efficiency, although the amplitude spectrum of the stimulus was left unchanged. On the other hand, a change in the filter bandwidth of the bandpass-filtered point stimuli affected efficiency only when it was also accompanied...
with a change in stimulus area. Thus, the decrease in detection efficiency was primarily due to the increase in stimulus area. When the detection efficiencies were plotted in semi-logarithmic coordinates as a function of stimulus area expressed in terms of the spatial spread of contrast energy ($\alpha$), the line of least squares explained 85% of the total variance.

Although an increase in stimulus area expressed in terms of the spatial spread of contrast energy ($\alpha$) explained the decrease of detection efficiency for the stimuli used in the current experiments, various grating studies and our previous studies with more complex stimuli indicate that the primary determinant of detection efficiency is the number of details in the image expressed by means of image complexity ($\alpha f_c^2$), i.e., the spatial spread of contrast energy ($\alpha$) multiplied by the center spatial frequency ($f_c$) squared. Thus, an increase in the number of details in an image results in an increase in the proportion of inefficiently collected contrast energy and consequently produces a decrease in efficiency. This finding is in agreement with the suggestion that the collection of information in the visual system is limited to a window of attention or sampling aperture and that the span of attention in each glimpse is limited to a relatively small amount of information.

However, for the point stimuli and circular grating of this study, image complexity ($\alpha f_c^2$) and spatial spread of contrast energy ($\alpha$) were equally good determinants of efficiency because the center spatial frequency for all the stimuli used was approximately constant (3.7 to 4.8 c/deg). On the other hand, when comparing the shapes of the detection efficiency curves as a function of image complexity for our present and previous studies, we found that they differed from each other.

As Figure 5 shows, efficiency at higher values of image complexity decreased faster for the randomized stimuli of the present study than for the well-structured stimuli of the previous studies, where the stimuli used were symbols (K, H, O, +) with various...

**FIGURE 4.** Detection efficiencies for the point stimuli and circular grating as a function of the spatial spread of contrast energy ($\alpha$). (A) Shows the results for HK, and (B) shows the results for RN. The least-squares curve fitted to the data is $\eta = 0.44 \times e^{-1.1\alpha}$.

**FIGURE 5.** Detection efficiencies for the stimuli of our current (open symbols) and previous (closed symbols) studies plotted as a function of image complexity ($\mathcal{Z} = \alpha f_c^2$) in double-logarithmic coordinates. The results shown are from subjects HK and RN. The previous data follow a potency function of $\eta = 0.39\mathcal{Z}^{-0.47}$, whereas the curve of least squares for our current data is $\eta = 0.46 \times e^{-0.071\mathcal{Z}}$. 

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bandwidths and ordered phase spectra, as well as simple and patched cosine gratings with various areas. The efficiency of spatial integration seems to be different for irregular patterns with random-phase spectra than for clearly structured stimuli with ordered-phase spectra.

In summary, our results indicate that when the center spatial frequency is kept constant, the main determinant of detection efficiency is stimulus area expressed in terms of the spatial spread of contrast energy. Stimulus bandwidth affects detection efficiency only if it changes the stimulus area.

Key Words
spatial integration, noise, detection efficiency, phase spectrum, human image processing

References