The physics of pulsatile blood flow with particular reference to small vessels

E. O. Attinger

Theoretical aspects of the analysis of pulsatile blood flow are discussed in the first part of the paper. In the second part the physical characteristics of the vascular system are described, and in the last part the behavior of the system is analyzed in terms of the outlined theory.

Following the work of Frank1 and Witzig2 early in this century, an increasing number of attempts have been made during the last decade to increase our understanding of pulsatile blood flow by applying analytical rather than purely descriptive methods for its investigation.3-6 While for a number of biological systems appropriate theories were already available from the physical sciences, the situation in the cardiovascular field was much more complicated. The theoretical hydrodynamicists concerned themselves primarily with ideal fluids and were never particularly interested in pulsatile flow of real liquids through pipes. The general theoretical development in their field was limited primarily by the fact that nonlinearities and instabilities are so ubiquitously present in fluid flows. As a consequence, the practical applications of hydrodynamics are based primarily on empirical relationships rather than on theoretical predictions. The situation in the rapidly developing field of rheology is similar. Because of its practical importance, large efforts have gone into the study of stress-strain and pressure-flow relations of various materials such as plastics and wood, and of suspensions. The similarity of the problems in rheology, hydrodynamics, and cardiovascular physiology has resulted in a rather extensive exchange of ideas between these different disciplines, beginning with Poiseuille, a physician interested in blood flow through capillaries, who first described laminar flow through tubes and stimulated much of the later theoretical work.

The analysis of a system aims at a quantitative description which permits the prediction of its behavior under a variety of circumstances. This behavior is considerably more complex in biological than in man-made systems. In order to make an analytical approach to a biosystem at all possible it becomes necessary to introduce simplifying assumptions, the judicious selection of which represents one of the most critical points of departure for any analysis and requires considerable insight into
biological phenomena. No matter how sophisticated the theoretical approach, its validity cannot be established by argument alone; experimental evidence is necessary to support it, and here we find ourselves faced with a major problem. The primary data required for such an analysis are pressure, flow, and volume. These variables change in pulsatile flow both with time and with location within the system. At many sites they are not measurable without seriously disturbing the system and even if the best available equipment and utmost care are used, measurement errors in the order of 5 per cent are unavoidable. Since the effects of some of the parameters which have been introduced in various analytical treatments are of the same order of magnitude, they cannot be assessed with any real confidence.

The first part of this paper deals with the theoretical aspects of the analysis of pulsatile blood flow. In the second part the physical characteristics of the vascular system are described and in the last part the behavior of the system is analyzed in terms of the outlined theory. For the sake of brevity, only the more significant aspects of the various problems are discussed; for more complete and detailed discussions the reader is referred to two recent publications.7, 8

Theoretical aspects

We begin with Newton's second equation:

Force = mass x acceleration,

which we apply to a unit volume of an incompressible Newtonian liquid in a cylindrical, elastic vessel segment under conditions of laminar flow (Fig. 1). To get a picture of laminar flow, imagine that the flowing liquid is composed of an infinite number of concentric layers, each forming a sleeve around the layers inside it. The outermost layer is in contact with the vessel wall and moves the same way as the wall does. Let this velocity be zero. The adjacent layer has a finite velocity, the next moves even faster, and so on. In the center of the tube, the velocity is largest. This velocity difference between adjacent layers produces shear forces, and the fluid elements within are subjected to shear strains. The ratio between this shearing stress (velocity gradient) and the rate of strain is called the coefficient of viscosity ($\mu$). For a Newtonian liquid, the relation between stress and rate of strain must satisfy two conditions; The coefficient of viscosity is constant; and if stress is zero, the rate of strain is also zero. In other words, a plot of these two variables must yield a straight line going through the origin.

Now consider the motion of fluid in this vessel segment. The pertinent dimensions of the vessel are: $R_o$, outside radius; $R_i$, inside radius; $h$, wall thickness; and $l$, the length along the z-axis. The fluid element moves with a velocity, $V$, which has two components, one parallel to the z-axis ($V_z$) and one parallel to the r-axis ($V_r$). Similarly, the pressure gradient $\nabla P$, which provides the net driving force for the element, also has two components ($8P/8z$ and $8P/8r$). (Since the vessel has a circular cross section, the third components in the tangential direction $V_\theta$ and $8P/8\theta$ are eliminated on the basis of symmetry.) According to Newton’s equation the total pressure gradient is equal to the total acceleration of the fluid element minus the force necessary to overcome viscous resistance, as shown in the upper equation of Fig. 1. In steady flow the acceleration term disappears, and the only force necessary to maintain the motion of fluid is that required to balance the viscous resistance of the liquid. Under these conditions there is no motion of the elastic vessel wall and the relation between flow and pressure reduces to the familiar equation of Poiseuille:

$$Q = \frac{\pi r^4 \Delta P}{8 \mu l}$$

In pulsatile flow the vessel wall moves during each cardiac cycle: during systole the radius increases, during diastole it decreases. We therefore have to write not
only two equations for the fluid motion (one in the direction parallel to the vessel axis and one in the radial direction) but we also have to consider the forces involved in the movement of the vessel wall itself (Panel C in Fig. 1). The force which causes this motion is the pressure gradient between the inside and the outside of the vessel (transmural pressure). The ratio between transmural pressure and the resulting deformation of the wall is called the elastic modulus $E$.

\[ E = \frac{\text{stress}}{\text{strain}} = \frac{P}{\Delta x/x_0} \]

where $\Delta x/x_0$ is the relative change in one dimension, say length. Since the vessel wall is three dimensional, there are three elastic moduli, one relating pressure to change in radius (tangential modulus), one relating pressure to change in length (longitudinal modulus), and one relating pressure to the change in wall thickness (radial modulus). These three moduli are related by six Poisson ratios ($\sigma_{ij}$), where Poisson ratio is defined as

\[ \sigma_{ij} = \frac{\text{strain in a direction i at right angles to the stress}}{\text{strain in the direction j of the stress}}. \]

---

**Fig. 1.** Diagram to illustrate the equations of motion for a fluid and a wall element in an elastic tube.
For example, the Poisson ratio for a stretched rubber band would be

$$\sigma_{\text{P}} = \frac{\Delta w/w_0}{\Delta l/l_0}$$

where $w_0$ = the initial width

$l_0$ = initial length

$\Delta w$ = change in width

$\Delta l$ = change in length

For isotropic materials (which vessel wall is not!) the elastic moduli are the same in all directions, and if the volume of the material does not change under stress, there is only one Poisson ratio with a value of one half.

According to Newton's third law there must be a force which counterbalances the excess pressure inside the vessel. This force is the tension developed by the elastic, collagenous, and muscular elements of the vessel wall. The relation between pressure ($P$) and wall tension ($T$) was first given by Laplace:

$$T = \frac{Pr}{h}$$

where $h$ = wall thickness.

As it turns out, the physical properties of the vessel wall (elasticity, viscosity, and inertia) are frequency-dependent, and the static elastic modulus is somewhat lower than the modulus at the frequency of the heartbeat. Except for the addition of a term which characterizes the elastic forces of the vessel wall, the form of the equations for wall motion is identical with those for the motion of the liquid. However, the equation of wall motion is expressed in terms of wall displacement, since we are primarily interested in the relation between stress and strain. The equation for fluid motion, on the other hand, is written in terms of velocity because here the relations between stress and rate of strain are most pertinent. The equations for the motion of fluid and the motion of the wall are inseparably linked. At the boundary between the blood and the vessel wall they must hold simultaneously. Furthermore, the peripheral vascular resistance, or better, impedance, which varies inversely with the fourth power of the radius, is highly sensitive to a change in transmural pressure.

Womersley and others have solved the Navier-Stokes equations for pulsatile flow in a uniform, elastic tube, and this solution is the basis of much of the recent work on arterial hemodynamics. (For excellent discussions of Womersley's theory and the assumption upon which his solution is based, see McDonald and Fry.) For an assumed pressure gradient $\Delta P = M \cos (\omega t - \phi)$ the solution for volume flow is

$$Q = \frac{\pi r^4 M}{\mu a^2} \sin (\omega t - \phi + \epsilon)$$

where $a^2 = \frac{\omega}{v}$

$\omega$ = radian (angular) frequency

$v$ = kinematic viscosity (viscosity/density) of the blood.

$M'$ and $\epsilon$ are basically the ratio and the phase angle difference of two Bessel functions of complex argument, and depend on wall thickness, Poisson ratio, and longitudinal tethering of the vessel.

As $\omega \rightarrow 0$, $M' / a^2 \rightarrow 1/6$ and $\epsilon \rightarrow 0$, i.e., the solution is that of Poiseuille.

Streeter has solved the equation of motion for pulsatile flow using the method of characteristics. In order to fit his experimental to his theoretical data he used different coefficients of viscosity for forward and backward flow because he assumed different conditions in the boundary layer and flow patterns under these two conditions. However, further experimental data are necessary to evaluate the relative merits of this approach.

In considering these equations and their solutions, we have to remember that the different variables are functions of space and time. The parameters of the system cannot, therefore, be treated as lumped (concentrated at one point), but must be considered distributed throughout the system. This is most easily demonstrated if one looks at the deformation which oc-
curs both in the pressure and in the flow pulse as they travel from the heart toward the periphery. The vascular system exhibits a highly complex geometry and the relative proportion of its structural components varies markedly throughout the vascular bed. As a consequence, wave reflections as well as selective damping are introduced, leading to the striking dispersion of pressure and flow pulses. It is apparent that the theoretical analysis cannot be applied to the cardiovascular system without reducing arbitrarily the complexity of the biological system.

The choice of these simplifying assumptions depends very much upon the physical properties of the system, which are briefly outlined in the next section.

Physical properties of the vascular system

A. Geometry. Fig. 2 shows a simplified diagram of the system. It can roughly be divided into four parts:

1. Two pumps in series, the left and right heart.
2. A distributing system, the arteries, leading from each ventricle into the periphery.
3. An exchange system, the capillaries, where metabolites diffuse across the capillary membrane both from and into the tissue.
4. A collecting system, the veins, which transport the blood back to the pump.

Note that the peripheral circulation is arranged in a number of parallel beds, each perfusing an organ system and fed from a common pump. It is obvious that the distribution of flow between these various beds will depend on their respective impedances. At rest about 10 per cent of the total cardiac output flows through the heart, 15 per cent through the brain, 25 per cent through the splanchnic bed, 25 per cent through the kidneys, 15 per cent through the muscles, and 10 per cent through the skin. Similarly, there are a number of parallel pathways through the lungs. Here, however, we deal with a low-pressure system, and hence the distribution of blood flow through the various lobes will depend primarily on the hydrostatic pressure difference, and therefore upon the position of the animal. During exercise, after meals, or in hot or cold environment these proportions may change considerably. For instance, the fraction of the cardiac output going to the eye or the heart of the submerged duck increases fourfold as compared to that in the non-submerged state, while the flow to the pancreas or the kidneys falls to one-tenth of its previous value. Hence, it is well to remember that the blood supply to an organ depends not only on the cardiac output but also on the state of the local vasculature.

As discussed in the previous section, the geometrical basis for an analytical treatment of pulsatile blood flow is a single, circular tube. Fig. 3 shows how large an approximation this represents. The data are based upon the values calculated by...
Fig. 3. Geometry of peripheral vascular tree of the dog. Diagram illustrating the massive changes in total cross section along the peripheral vascular bed. Blocks are numbered as follows: 1, capillaries. 2a, arterioles. 2b, veinules. 3a, terminal arterial branches. 3b, terminal veins. 4a, main arterial branches. 4b, main venous branches. 5a, large arteries. 5b, large veins. 6a, aorta. 6b, vena cava. Resistance values pertain to the total effect of one segment.

Green for a 13 kilogram dog. A single tube, the aorta, with a cross section of about 0.8 cm.² branches progressively into $12 \times 10^5$ capillaries, each of which has a cross section of about $5 \times 10^{-7}$ cm.² The total cross section of the capillaries represents an area of about 600 cm.². Hence, for a 13 kilogram dog with a cardiac output of 1.8 L. per minute or 30 cm.³ per second, the average linear velocity of the flowing blood decreases from 37.5 cm. per second in the aorta to 0.05 cm. per second in a capillary. Since blood is a non-Newtonian liquid, the viscosity of which falls as the rate of strain (velocity) increases, particularly at high hematocrits, the pressure-flow relations in the various segments can be expected to be different on this basis alone. Note, however, that at any one time only a fraction of the total capillary bed (60 to 75 per cent) is perfused.

Although the total cross section increases, the cross section of an individual artery decreases during its course toward the periphery. As an example, the mean radius of the aortas of ten large dogs was found to vary from 1.2 cm. in the ascending aorta to 0.75 cm. in the middle of the descending aorta, to 0.5 cm. in the abdominal aorta. The corresponding radius in the external iliac artery was 0.27 cm. Over the same distance the elastic modulus of the arterial wall increases from $3 \times 10^{10}$ dyn. cm.⁻² to $12 \times 10^{10}$ dyn. cm.⁻², i.e., the wall material of the arteries is about four times stiffer in the periphery as compared to the central part of the aortic tree.

On the venous side the system decreases from the total cross section of the capillary bed to that of the vena cava. Here, data on the physical characteristics of the individual segments are even scarcer than on the arterial side. It is, however, reasonable to assume that this system also is nonuniform along its length. Furthermore, because it is a low pressure system, its dimensions vary considerably more with pressure changes than those of the high pressure system.

B. Pressure-volume relations. The stress-strain relations with respect to linear dimensions have been discussed in the previous section. Their over-all effect determines the instantaneous volume of a particular vascular segment. The latter can also be described by the vessel distensibility, $C$, which in terms of vessel dimensions and elastic modulus becomes:

$$\frac{dV}{dP} = \frac{3\pi r^2(a+1)^2}{E(2a+1)}$$
where $\frac{dV}{dP}$ is the change in volume per unit pressure change $dP$, and $a$ is the ratio between radius and wall thickness. In the arterial system, the mean distending pressure is in the order of $130 \times 10^3$ dyn. cm.$^{-2}$ (100 mm. Hg) upon which is superimposed the oscillating pulse pressure. Since the arteries are quite stiff ($E = 3$ to $12$ dyn. cm.$^{-2} \times 10^4$) this pressure change results in a change in radius of only 2 to 5 per cent. This rather simplified analysis is based upon the assumption that the vessel length does not change as the transmural pressure changes. In reality, the dimensions of the vessel change not only in the circumferential direction, but also along the axis and in wall thickness. Hence there would be 3 moduli of elasticity to consider which are related for an anisotropic, nonisovolumetric material by six Poisson ratios. Usually isovolumetry and a single Poisson ratio of 0.5 are assumed, but, although this assumption has been proved to be incorrect, the effects upon vascular dynamics are not yet measurable in the in vivo system. While the available evidence indicates that the elastic properties of the arterial tree are linear over the normal operating range (100 to 200 dyn. cm.$^{-2} \times 10^3$) nonlinear effects are considerably more important in the venous system.

As the volume of a completely collapsed vein is increased, the vessel cross section changes from a very flat ellipse to a circle without initially producing a wall tension. Once wall tension (and hence a transmural pressure) begins to develop, the rise in pressure per unit volume change is at first rather small but increases rapidly thereafter. For instance, Green's data indicate that for the inferior vena cava of a dog the distending pressures are as follows: for a volume of $2$ cm.$^3$, $P = 0$, for $V = 8$ cm.$^3$, $P = 2$ cm. H$_2$O, for $V = 12$ cm.$^3$, $P = 12$ cm. H$_2$O, and for $V = 14$ cm.$^3$, $P = 24$ cm. H$_2$O). Hence, the distensibility varies from practically infinity at a volume of 0 to 2 cm.$^3$ to $0.166 \times 10^{-3}$ cm.$^3$ dyn.$^{-1}$ at a volume of 14 cm.$^3$.

Of the total blood volume, about 22 per cent resides in the pulmonary circulation, 14 per cent in the heart, and 64 per cent in the peripheral circulation. The distribution among the vessels of the latter is as follows: about 17 per cent is found in arteries with a diameter greater than 0.3 mm., about 17.8 per cent in the microcirculation (radius less than 0.05 mm.), and 64 per cent in the venous bed. Based on geometrical considerations, as well as on pressure-flow and pressure-volume relations, we have estimated the volume distribution in the different parallel beds of the peripheral circulation as follows: splanchnic bed 38 per cent, muscle 21 per cent, brain 17 per cent, skin 14 per cent, kidney 7.1 per cent, coronary circulation 3 per cent. The splanchnic bed can absorb or release roughly 40 per cent of its normal volume; similar relations hold for the vascular bed of the skin and muscle.

C. Pressures and flows in the vascular bed. The total change in mean pressure across the peripheral vascular bed is in the order of 100 mm. Hg ($130 \times 10^3$ dyn. cm.$^{-2}$). The main pressure drop (65 per cent) occurs through the arterioles (mean diameter 0.02 cm., mean length 0.2 cm.). In contrast, the fall in mean pressure along the large arteries is very small, although the pulse pressure increases. The pressure drop across the capillaries will depend greatly on the state of the arterioles. If they are dilated, their contribution to the total pressure loss will be less and that of the capillaries will increase. It is good to remember that the control of the capillary pressure, and of the gradient across the capillary bed is a major factor for the control of fluid exchange across the capillary membrane. The fall in pressure along the venous system represents only about 10 per cent of the total pressure loss. These relations are similar for both the peripheral and the pulmonary circulation. One can calculate that a reduction of the cross section of the capillary bed by one half increases the total resistance across the peripheral vascular bed by about 30 per cent;
if half the arterioles are blocked, the resistance increases by 60 per cent, and so on. During its travel the pressure wave changes considerably in shape (Fig. 4). The amplitude of the pressure pulse increases, the front of the wave becomes steeper, and the sharp inflection at the incisura is smoothed out and then disappears entirely by the time the wave reaches the lower abdominal aorta. The time required for the pressure pulse to travel from the heart to the femoral artery is between 50 and 100 msec. Between these larger arteries and the small arteries with an internal diameter of 200 μ, the mean blood pressure falls by 10 to 15 per cent and the pulse pressure decreases. The pressure is still markedly pulsatile and, although it is dampened somewhat, the available evidence indicates that the main pressure drop occurs in the arteries of less than 200 μ internal diameter.²⁹

Flow in the large arteries is markedly pulsatile. In the ascending aorta of the dog, the average volume flow is in the order of 30 to 40 cm³ per second. The peak

Fig. 4. Change in the shape of a pressure wave traveling from the heart toward the periphery. The site of measurement and its distance from the aortic arch are marked beside each pressure pulse. Note the time delays between the different measuring sites.
velocity during systole may reach 100 to 200 cm. per second. The ejection period is followed by a short interval during which blood actually flows backward. Over the remainder of the cardiac cycle the velocity is zero. Toward the periphery this highly peaked flow pattern is progressively dampened out, backflow disappears, and forward flow is present over the whole cycle. Both the pattern and the distribution of flow depend markedly on the characteristics of the particular vascular bed. In arteries of 1 mm. diameter the pattern is still quite oscillatory and available evidence suggests that the pulsatile flow character may be preserved even in pulmonary capillaries. On the venous side the flow in the inferior and superior vena cava is strongly pulsatile. Backflow may be present in the latter where the magnitude of the oscillatory components is of the same order as that of the mean flow. No reliable data are yet available on the dynamic flow patterns in the other parts of the venous vasculature.

As far as the blood flow through the eye is concerned, the average flow rate per 100 Gm. of retina is in the order of 500 cm.³ per minute. This represents a total flow of about 0.5 cm.³ per minute for the cat eye. About 20 per cent flows through the retinal vessels, the remaining 80 per cent through the chorio-capillaries. The latter supply the nutrient flow for a tissue with an extremely high metabolism (QO₂ = 31). Direct observation of the microcirculation of the retina indicates that blood flow in the retinal vessels is relatively steady and that all the capillaries appear to be perfused.

D. The physical characteristics of blood.
Since blood represents a suspension of cells in a colloidal solution its behavior is not that of a Newtonian liquid. The values given for the viscosity of plasma are generally about 1.39 centipoise and those for whole blood between 1.7 and 3.7 cp. at 37°C. The viscosity at low rates of shear is considerably higher. Typical values obtained by Kunz and Coulter for oscillatory flow are listed in Table I.

Their evidence also indicates that the orientation of the red cells within the velocity field changes during the cardiac cycle. Close to the vessel wall there is a relatively cell-poor layer, the width of which fluctuates from instant to instant, decreases as the hematocrit increases and whose effects become relatively less important in larger vessels, where the flow approaches that of a homogeneous fluid. The higher velocities in the center in combination with a peripheral layer lead to core concentrations which are lower than if the velocity were uniform over the cross section. Such a dynamic effect is but one demonstration of the complex interaction of velocity and concentration distributions. At bifurcations the concentration of suspended particles is usually lower in the side branch and is affected primarily by the rate of discharge in the two branches, the branch size, and the concentration upstream. As the red blood cells pass from the arterioles into the capillaries their shape changes from that of an elliptic disc to that of a thimble.

The analysis of pulsatile flow
We are now in a position to interpret the behavior of the cardiovascular system within the background provided by the theory. The first problem in any analytical procedure is to express the measured variable as a number. A convenient way of doing this is the use of mean values, i.e., integration with respect to time or space, for example: cardiac output, minute ven-

Table I. Change in blood viscosity with peak flow rate in pulsatile flow

<table>
<thead>
<tr>
<th>Hematocrit (%)</th>
<th>Apparent viscosity (centipoise)*</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>100</td>
<td>70</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>60</td>
<td>25</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>68.5</td>
<td>18</td>
<td>12</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>7</td>
<td>6</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

*Column a, 0.25 cm.³ per second; column b, 1 cm.³ per second; column c, 4 cm.³ per second.
tilation, peripheral resistance, diffusion capacity, etc. However, for certain purposes this approach neglects significant characteristics. There are a number of mathematical techniques which make it possible to express any arbitrary function of time as a series of terms. For periodic and quasi-periodic phenomena, such as encountered in pulsatile flow, the so-called Fourier series are particularly suited. With this technique flow and pressure values are expressed as the sum of a number of sine waves, which permits a mathematical treatment of the data in terms of meaningful physical concepts. In such an analysis (Fig. 5) one finds that the amplitude of the various harmonics (frequency components) of the pressure pulse increases from the arch of the aorta both toward the fore and the hindpart of the body. 26 (This, of course, is in accordance with the over-all increase in pulse pressure illustrated in Fig. 4.) In this figure the distances from the aortic arch in centimeters are plotted on the abscissa and the amplitude of the first 4 harmonics (from top to bottom) on the ordinate. It will be seen that the increase in amplitude for the first 3 harmonics is rather steep until the inguinal ligament and that thereafter the amplitude falls off. Toward the head the increase occurs only in the first harmonic and is much smaller. The fact that the amplitude changes of the various harmonics are not parallel indicates that this behavior is not only frequency dependent. However, it clearly shows that the vessels become stiffer toward the periphery. Fig. 6 shows how the pulse wave velocity (foot-to-foot velocity) changes in the various arteries. Again distance in centimeters from the heart is plotted on the abscissa. At a mean pressure of 200 cm. H₂O the wave velocity in the aortic arch is approximately 4 meters per second. It increases to 7 m per second in the carotid, to 8 m per second at the bifurcation of the aorta and to 14 m per second in the arteries of the foot. 27 Since according to the Moens-Korteweg equation:

\[
\begin{align*}
(7) \quad c^2 &= \frac{Eh}{2\rho} \\
\end{align*}
\]

symbols as defined earlier),

![Fig. 5. Changes in the amplitude of the first (top), second, third, and fourth (bottom) harmonics of the pressure pulse, as the pressure pulse travels from the heart both cephalad and caudal into the periphery. Pressure measurements were obtained in intervals of 2 to 5 cm.; the distances are indicated on the abscissa. Note the marked increase in the first three harmonics as the wave approaches the inguinal ligament and the decrease thereafter. The increase toward the head is less striking.](image-url)
the wave velocity \( c \) increases proportionally to the square root of the elastic modulus, the expected increase in the latter would be respectively 1.3 fold in the carotid, 1.4 fold at the bifurcation, and 1.9 fold in the arteries of the foot, if the ratio wall thickness/radius remains constant. The change in the shape of the pressure wave shown in Fig. 4 and Fig. 5 can be explained by a combination of three mechanisms originating from the geometry and nonuniformity of the vascular wall: wave reflections, viscous losses, and elastic tapering.

A. Sudden changes in impedance are associated with wave reflections. At such points only a part of the pulse wave traveling peripherally is transmitted, the remainder is reflected back toward the heart. Since frictional losses occur, the amplitude of the incident wave decreases along the vessel toward the periphery and the amplitude of the reflected wave decreases toward the origin of the vessel. Because of these frictional losses no true standing waves can be produced, and there are probably no secondary reflections. At present the theory of wave reflections appears to give the best explanation for the peaks and troughs in the amplitude of the various harmonics of pressure and flow pulses. Although these patterns can be analyzed for descriptive purposes as the superposition of two waves, one has to think of the superposition of a number of waves, each reflection originating at a different site.

B. Losses are introduced because of the
viscous properties of the liquid and the wall. The viscosity of the blood is, of course, one of the important factors for the pressure drop across the vascular bed (Equations 1 and 5). It has been proposed that a frequency dependence of wall viscosity may account for the change in the shape of the pressure wave. However, available data from in vitro experiments do not support this suggestion. 28, 29 Our present measuring techniques are not accurate enough to evaluate the effects of wall viscosity on the transmission of the pressure wave in the living animal.

C. Elastic tapering, i.e., the progressive stiffening of the vessel wall toward the periphery has the following effects upon the over-all function of the cardiovascular system 30:

- It reduces the oscillatory component of the work required to maintain the cardiac output.
- It reduces the over-all distensibility of the arterial system.
- It amplifies the pressure pulse as it travels toward the periphery.

Since both flow and pressure are pulsatile, the concept of peripheral resistance has to be modified in order to account for the frequency-dependent behavior of the system. Instead of using only the ratio of mean pressure and mean flow, the pressure-flow relation has to be evaluated for all the pertinent frequencies. The frequency-dependent parameter obtained in this way is called the impedance, and is, of course, identical with peripheral resistance at zero frequency. Fig. 7 shows the input impedance of a 60 pound dog measured simultaneously in the ascending aorta, the descending aorta (mid-thoracic), the abdominal aorta (below the renal arteries), and the carotis communis. The oscillatory terms represent only a fraction of the DC impedance term. The impedance values are normalized with respect to the DC value. Note that the magnitude of the latter as well as that of the oscillatory components increases progressively toward the periphery.

While it is certainly true that a number of additional phenomena, such as the effects due to entrance length, non-Newtonian behavior of flow, and nonlaminar flow patterns will have to be included in a final analysis of the behavior of the large vessel, they appear to be too small to be readily detected with the measuring equipment presently available. Hence, it seems justifiable to consider the flow behavior in large vessels in terms of continuum mechanics, i.e., local variations at the microscopic level are replaced by statistical averages. This is not true for the microcirculation. 30 Here the detailed flow behavior of blood cannot be described in terms of bulk rheological parameters of blood. If we were to count the molecules of the various plasma components and the portions of red cells contained in a unit volume a large number of times, we could not only arrive at an average density of each component, but could also define a local average velocity. In larger vessels we can extend this time average to a space average taken at a single fixed time; however, in the microcirculation there is no volume element in the flow possessing this average property. Direct observation of the microcirculation indicates that the flow of plasma and erythrocytes is completely random both with respect to their relative proportions and with respect to direction along the vessel axis. I know of no theory which could adequately describe blood flow through the microcirculation in any detail.

Conrad and Green 32 have evaluated hemodynamic changes in peripheral vascular disease. The vasculature of normal individuals is characterized by a vasodilator response to alcohol, resulting in a marked increase in the flow rate, with respect to both the mean and the oscillatory components. The pressure drop through large arteries and the pulse wave velocity did not change but the resistance of the small vessels fell to one third of the control value.

In patients with vasospastic disease, the values for blood flow were considerably
Fig. 7. Magnitude of the input impedance in various vascular beds of a 60 pound dog as a function of frequency. Pressures and flows were measured simultaneously in the ascending, descending, lower abdominal aorta and in the common carotid. The impedance values are normalized. The value for the DC impedance is given in the inset.

Reduced. There was a significant increase in the pressure drop through the large arteries and the resistance through small vessels was markedly elevated. Pulse wave velocity remained unchanged. Although, following the administration of alcohol, marked improvement in vascular performance was observed, the values of the parameters measured remained in general below normal values of the controls. In arterial occlusion, the observed pressure drop through the large arteries (brachial to digital artery) was increased 80 fold in the average as compared to the normal. As a consequence, blood flow (both average value and oscillatory components) was greatly reduced. A rise in small vessel resistance (3 fold as compared to normal) and a decrease in pulse wave velocity was also found in these cases. The response of these parameters to vasodilation by alcohol was rather small.
Conclusion

It is apparent that the analysis of the pressure-flow relations of pulsatile blood flow in a complex vascular bed is not a simple matter. However, it is of considerable importance, both in terms of cardiac work and in terms of organ perfusion. Although the evidence is not clear-cut, it has often been suggested that an organ functions better if it is perfused by means of pulsatile rather than steady flow. Furthermore, pulsatile flow is not restricted to the arterial bed alone. In both the inferior and the superior vena cava blood flow is oscillatory. These oscillations are probably due primarily to back flow associated with atrial contraction. However, the study of the dynamic behavior of blood flow in veins has hardly begun.

The frequency-dependence of vascular impedance suggests that there is an optimal rate at which cardiac work is minimal for a given cardiac output. Calculations for the pulmonary bed of dogs show that the total power dissipated falls from 300 mwatts at a heart rate of 30 per minute to 100 mwatts at 180 per minute.93 The power losses due to the pulsatile components, therefore, can represent a considerable fraction of the total power dissipation. Furthermore, the distribution of flow and volume between the various vascular beds is a frequency-dependent phenomenon. It will be of great interest to carry this analysis into the smaller vessels, once adequate data about their dimensions and pressure-flow patterns become available. The ophthalmologist is in the enviable position of being able to provide such data for the small vessels in the fundus of the eye. By careful microphotographic techniques he can evaluate not only the flow pattern, but the physical properties of the vessel wall as well, using the pulse wave velocity as an index for the latter.

I am grateful to Dr. D. A. McDonald of the Research Institute of the Presbyterian Hospital in Philadelphia for allowing me to use some of his unpublished material (Figs. 4 to 6).

REFERENCES