The trabecular mesh: a mathematical analysis

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The corneoscleral trabecular meshwork is a three-dimensional net of diagonally crossing collagenous fibers invested in endothelium. Tension on the mesh should cause it to tend to bow inward and hold Schlemm's canal open. Inward bowing is restrained by attachment of the outer mesh to the corneosclera, by meridional uveal mesh fibers, and by resistance of mesh element crossing to deformation. A lamella of meshwork is analyzed from a geometrical point of view, and it is found that even were the mesh free to assume an inward bowed hyperboloidal shape, the spur movement required to change mesh configuration from that of a cone frustum to hyperboloid would be only 0.047 mm and not readily recognizable gonioscopically.

Key words: trabecular mesh, scleral spur, Schlemm's canal, ciliary muscle

The framework of the corneoscleral trabecular mesh consists of a number of bundles of collagen fibers obliquely disposed so that each bundle spirals from the region of Schwalbe's ring across the internal scleral furrow to the scleral spur. To a first approximation the trabecular mesh forms a frustum of a right circular cone as shown schematically in Fig. 1. The oblique crossings of the bundles form a grid resembling a tube of cloth cut "on the bias," or a tube of woven wire used for electrical shielding, or the diagonally woven tubular Chinese finger puzzle. These latter forms neck inward when stretched lengthwise, suggesting that ciliary muscle tension on the trabecular mesh should cause the mesh to bow inward toward the eye axis. Since the mesh forms the inner wall of the canal, inward bowing of the mesh should tend to open Schlemm's canal. The present study gives a first-order estimate of the movement of the spur which would produce maximal inward bowing of the mesh.

The innermost lamellae of the corneoscleral trabecular mesh are attached to ciliary body and can be tensed by longitudinal ciliary muscle contraction. We have suggested that the intermediate layers of trabecular mesh are acted on by ciliary muscle tendons which pass within the ring of collagenous tissue that forms the scleral spur, the spur-ring acting as a sort of pulley. The outer lamellae of the corneoscleral trabecular mesh are visualized as pulled inward by numerous interlamellar attachments to the actively tensed layers, for they appear to have no ciliary muscle tendons inserting into them and movement of the firm base of the scleral spur-ring (the scleral roll) seems improbable.

Analysis

The present analysis is of a lamella of the trabecular mesh extending from the tip of the scleral spur to the sclera near Schwalbe's line. Professor Gustav Mesmer of the Department of Mechanical Engineering of Washington University...
Fig. 1. A cone is fitted into the anterior segment of the eye so that Schwalbe’s ring and the tip of the scleral spur form circles on the cone surface. The trabecular mesh fibers spiral on the surface of the cone from scleral spur to Schwalbe’s ring.

Aided in formulating the geometrical model. In its “relaxed” state, each element or “beam” of the mesh lamella is assumed to spiral on the surface of a cone from the spur to near Schwalbe’s ring just as elements of the Chinese finger puzzle wind around the surface of a cylinder. Figs. 2 and 3 illustrate the conical model. If the lamella were a free conical tube and all fibers of this lamella were tensed to straight lines in space, a single beam or cord of the mesh would then be, as in Fig. 4, a cord stretched obliquely from a point on the corneosclera near Schwalbe’s ring to a point on the tip of the scleral spur. If the stretched mesh element is repeated around the eye (rotated around the geometric axis), it forms a hyperboloid of revolution of one sheet, the profile of which in a meridional plane of the eye is a hyperbola (Fig. 5).

The length of an element of the trabecular mesh and its points of attachment near Schwalbe’s ring and the spur may be approximated from published or other readily available material by the following means.

The central section of a set of serial sections of a normal adult eye was photographically enlarged and measured. The results (after reduction to actual size) establish the radii of the mesh attachments at Schwalbe’s ring, $R_1$, and spur, $R_2$, and the distance between them, $h$, (Fig. 6). The results of our measurements are $R_1 = 5.431$ mm, $R_2 = 5.800$ mm, $h = 0.369$ mm. Thus the cone on which the points chosen lie is a right circular cone, the slant of which makes an angle of $\Omega = 45^\circ$ with the cone axis (the geometric axis of the eye).

The weavelike structure of the trabecular mesh is well demonstrated in various publications (e.g., refs. 5 to 7 and in Fig. 7). The angle between the mesh fibers and the scleral spur can be estimated from the photographs, but the angle between the geometrical axis of the eye and the plane of section is not given in the publications we have examined. It is assumed therefore that the plane of trabecular mesh section is also 45° to the geometric axis of the eye and that the predominant mesh fibers seen in the illustrations lie in the layer of trabecular mesh under consideration. In the illustrations one sees trabecular mesh fibers which make a spectrum of angles with the spur, from about 10° to 20°. We have taken the intermediate value of 15° between mesh element and spur as representative.

From the knowledge of the distance between the anterior attachment point and the spur tip in the "tangent" plane and from the angle that a mesh element makes with the spur, we construct a cone with a mesh element spiraling on its surface from spur to near Schwalbe’s line.

The geometrical model used in this analysis of a mesh fiber is that of a segment of a constant-pitch conical spiral defined by equation 1 and illustrated in Fig. 2. By constant pitch we imply that for every 2 $\pi$ radian increase in the angular coordinate $\theta$, the vector $r$ defining a point on the spiral grows by a constant amount, $k$. The segment of interest in this analysis crosses two circles of radii $R_1$ and $R_2$ corresponding to Schwalbe’s ring and the scleral spur, respectively.

$$ r = k \theta $$

From equation 1 we see that the spiral is completely defined by the parameter $k$. This spiral parameter was derived from measurement of $\alpha$, $\beta$, and $\gamma$. The result is $k = 0.0056$. The segment of interest is then a constant-pitch conical spiral of radius $k \theta$.
Fig. 2. Cone in side view. A mesh element is seen spiraling on the surface of the cone between scleral spur and Schwalbe’s ring as a thick line, and the continuation of the course of the element on the cone surface is traced in a thin line. 2θ, Total apical angle of the cone; r, distance of a point on the spiral from the apex; R, radius of the cone at any point; α, angle between the mesh element and scleral spur as measured on the cone surface; h, axial distance between Schwalbe’s ring and scleral spur; dS, differential element of the spiral.

Fig. 3. Cone and mesh spiral seen in end view. R1, Radius of Schwalbe’s ring; R2, radius of the spur at its tip; θ, angle of rotation of the spiral about the cone axis from the cone apex to any point on the spiral.

the angle at which the spiral crosses the circle on the cone at radius R. It may be shown that at any radius, R, and corresponding crossing angle α

\[ \tan \alpha = \frac{k}{R} \]

or

\[ k = R \tan \alpha \] (2)

By the coordinate system shown in Figs. 2 and 3, a differential element, dS, on the spiral is given by the equation

\[ dS = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2 \sin^2 \Omega} \ d\theta \]

Substituting into this relation equation 1, the defining relation of the spiral, we obtain

\[ dS = k \sin \Omega \sqrt{\frac{1}{\sin^2 \Omega} + \theta^2} \ d\theta \] (3)

The total segment length from radius R1 to radius R2, S_{12}, with associated angular coordinates θ1 and θ2, is obtained by integration of equation 3 to give

\[ S_{12} = \frac{k \sin \Omega}{2} \left[ \theta \sqrt{\theta^2 + \frac{1}{\sin^2 \Omega}} + \ln \left( \theta + \sqrt{\theta^2 + \frac{1}{\sin^2 \Omega}} \right) \right]_{\theta_1}^{\theta_2} \] (4)

Now since

\[ \theta = r/k \] (5)

and

\[ R = r \sin \Omega \] (6)

the angles θ1 and θ2 are determined by combining equations 5 and 6 to get

\[ \theta = \frac{R}{k \sin \Omega} \]

where k is determined by photographic measurements of R and α as previously outlined.

One other geometric parameter remains to be defined before S_{12} may be evaluated from equation 4. That parameter is the cone half-angle Ω. Like the spiral parameter k (which is independent of the cone angle), this quantity has been determined from photographic measurements as described in a previous section.

The second part of this analysis deals with the following question. If one were to move the scleral spur, R2, axially away from the apex of the cone without rotation until the segment S_{12} was pulled...
Fig. 4. Single mesh element is seen spiraling on the surface of a cone from the scleral spur to Schwalbe’s line (solid lines). When the scleral spur is displaced posteriorly by a distance $\Delta h$, the mesh element is pulled into a straight line in space (interrupted lines).

Fig. 5. Mesh elements have been pulled from spirals to straight lines in space; their profile forms a hyperbola. The elements at the right side of the drawing have been elongated to show the hyperbola more clearly. A mesh element lying parallel to the plane of the paper runs from Schwalbe’s line (point 1) to the spur (point 2). The line through this element is one asymptote of the hyperbola; the line through a similar element behind the plane of the page $(1', 2')$ is the other asymptote.

By choice of the coordinates, $x_1$ is zero, so that

$$h_z^2 = S_{1z}^2 - x_z^2 - (y_z - y_1)^2$$

where $h_z$ is the new axial separation of $R_1$ and $R_z$.

Finally, the axial displacement is determined from the following:

$$h_z = R_1$$

$$y_z = R_2 \cos \Delta \theta$$

$$\Delta \theta = \theta_2 - \theta_1$$

$$x_z = R_2 \sin \Delta \theta$$
Fig. 6. Anterior chamber angle traced from an enlargement of meridional section of a human eye (solid lines). C, cornea; SC, Schlemm's canal; S, scleral spur; I, iris. The interrupted lines show the calculated position of the scleral spur and trabecular mesh after traction on the spur has pulled the mesh from conical to hyperboloidal. $R_1$ and $R_2$, Radii of Schwalbe's ring and spur; $h$, distance from Schwalbe's line to spur (mesh in conical form); $\Delta h$, distance the spur was moved to change mesh from conical to hyperboloidal.

$$\Delta h = h_2 - h_1$$

where $h_1$ is the original axial separation of $R_1$ and $R_2$ before straightening of the segment took place (Fig. 2).

The length of a trabecular mesh element spiraling from spur to Schwalbe's line is estimated from equation 4 to be 1.956 mm, and the movement of the spur necessary to straighten the element, $\Delta h = 0.047$ mm.

If the spur ring were to be pulled posteriorly without changing diameter and if the mesh elements in the lamella under consideration could be pulled to straight lines in space (Fig. 4), the shape of the extended lamella would be a hyperboloid and its profile as seen in meridional section would be a hyperbola (Fig. 5). The derivation of the equation of the hyperbolic profile of the tensed mesh lamella involves finding the equation of a mesh element. It is convenient to locate Schwalbe's ring in the x-y plane, with the geometric axis of the eye as the z axis and the mesh element in a plane parallel to the x-z plane. The equation of the element projected onto the x-z plane is, then, that of one asymptote of the hyperbola. The equation of the element after 180° rotation about the z axis is the other asymptote. The distance of the element from the z axis is the transverse semiaxis, $a$, of the hyperbola symbolized by

$$\frac{x^2}{a^2} - \frac{(z - q)^2}{h^2} = 1$$

In this equation, $h$ represents the conjugate semiaxis of the hyperbola and $q$ the displacement of the hyperbola in the z direction (Fig. 9).

For our set of constants, the hyperbola is

$$\frac{x^2}{(5.429)^2} - \frac{(z - 0.0279)^2}{(1.181)^2} = 1$$

The spur movement and resulting hyperbola are drawn to scale in Fig. 6.

Discussion

It is of interest to analyze the work of Grierson et al. in the context of the present paper. These authors examined two groups of
eyes surgically enucleated and immediately placed in fixative. One group had received repeated doses of pilocarpine before enucleation, whereas the other group had been untreated. In histologic sections the angle between the scleral spur and the sclera was measured; we call this angle $\gamma$. It was found that the spur was pulled backward in the pilocarpine-treated eyes. If the spur swings on its scleral attachment as on a hinge, one can approximate the position of the spur tip from the measurements given, the length of the spur, and the angle of the scleral surface to a frontal plane. We found (see above) that a line through the tip of the spur and Schwalbe's line made an angle of 45° with the axis of the eye and hence 45° with the frontal plane. If we assume that this is parallel to the inner surface of the sclera from which Grierson et al. measured, then the angle between the spur and the frontal plane is (45° - $\gamma$). We take the frontal plane through the base of the spur. If the length of the spur is $l$, then the distance, $h$, from the spur tip to the frontal plane is $h = l \sin (45° - \gamma)$, and the distance $d$ (in the frontal plane) from the spur base to the spur tip is $d = l \cos (45° - \gamma)$. Nesterov et al. give $l$ as 0.0895 mm, but we suspect that he meant this to be the width of the spur. From our sections we estimate the length of the spur as 0.150 mm.

Using $l = 0.150$ and the means of Grierson et al. for $\gamma$, we find the following values:

**Untreated:**
- $\gamma = 16.8°$
- $h = 0.071$ mm
- $d = 0.132$ mm

**Treated:**
- $\gamma = 31.1°$
- $h = 0.036$ mm
- $d = 0.146$ mm

$\Delta h = 0.035$
$\Delta d = 0.014$

Thus pilocarpine caused the spur tip to move posteriorly about 0.035 mm compared to the maximum posterior displacement of 0.047 mm we have calculated for a single trabecular lamella without restraint. The figures are in satisfying concordance. According to the calculations in the list, in swinging backward under the influence of pilocarpine, the spur tip also swings inward 0.014 mm. We doubt that this inward motion occurs, for reasons we have detailed in ref. 1.

Clearly, the trabecular mesh does not assume a grossly hourglass shape on contraction of the ciliary muscle. The following three anatomical relations restrict changes in the mesh:

1. Probably the most important of these is that the sheets of fibers or lamellae of the mesh are oversimplifications in the sense that
the mesh actually is a sponge-like structure with frequent bands lacing the lamellae together. As Hogan et al.\(^2\) say on page 155:

"The corneoscleral meshwork is composed of a large number of flattened perforated sheets which branch and interlace with each other. The branches may lie in a single plane or connect to more superficial or deeper sheets . . . The corneoscleral sheets, therefore, form a circular, three-dimensional grid within which are the intertrabecular spaces." This is beautifully illustrated in Fig. 10 (Fig. 4-29, page 153) taken from the same text. The external aspect of the spongy mesh is fixed to the corneosclera except for the region of Schlemm's canal, thus restricting inward bowing of the mesh.

2. A few fibers of the uveal trabecular mesh run meridionally forward. Such fibers should restrict inward bowing of the mesh.

3. Adhesion of trabeculae at crossings and investment of the crossings in endothelium restricts the change in crossing angle necessary to free alteration of mesh configuration.

In spite of these restrictions, ciliary muscle tension on the mesh should cause inward bowing toward the hyperboloidal shape predicted for a single unrestrained lamella of mesh.

A change in the contour of the trabecular mesh as a result of ciliary muscle contraction as in accommodation or after administration of pilocarpine is not seen gonioscopically, nor is the spur seen to move. According to our calculations, these changes are too small to be detected under clinical conditions. We anticipate that in life, under the restraints mentioned above, spur movement would be still smaller than we have calculated.

The tendency toward inward necking of
the tensed trabecular sheets should open the canal of Schlemm and resist closure of the canal by intraocular pressure. The fact that the tendency to necking is greatest in the anterior portion of the mesh should aid in maintaining patency of the anterior portion of Schlemm's canal.

The restraint of inward trabecular bowing by meridional uveal trabecular mesh fibers appears to have particular importance in congenital glaucoma. Trabeculotomy in congenital glaucoma severs redundant meridional uveal trabecular mesh fibers and relieves their excessive restraint on mesh mobility. Chandler and Grant8 state, "The point of the knife is directed so as to engage the uveal meshwork where it overlies the anterior third of the corneoscleral trabecular meshwork . . . the tip of the knife incising uveal meshwork so that a gaping incision is visible, but trying to keep superficial enough to avoid injury to the underlying corneoscleral trabecular meshwork."

Tension on the trabecular mesh is primarily due to the elastic choroid. The tension is increased during contraction of the meridional portion of the ciliary muscle. Therefore it is to be anticipated that the mesh will become less tense and the canal more easily collapsed if the choroid becomes less elastic and the ciliary muscle less active, as perhaps occurs with age or as the result of cycloplegic drugs.

REFERENCES