A Photographic Technique for Measuring Horizontal and Vertical Eye Alignment Throughout the Field of Gaze

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We present a photographic method based upon corneal light reflections for the measurement of binocular misalignment. Our procedures allow for the measurement of eye alignment errors to fixation targets presented at any distance throughout the subject's field of gaze, and allow for the measurement of errors in the horizontal and vertical directions. Furthermore, estimates of the alignment state can be made simultaneously from both eyes while fixation targets are presented monocularly or binocularly. This photographic method represents an enhancement of typical clinical prism and cover methods because: (1) it can provide extensive information efficiently about patterns of misalignment across a large number of fixation locations; (2) it also can provide information about the scatter in addition to the magnitude of convergence error; and (3) it can be easily applied to noncooperative subjects such as animals and young children. Furthermore, the procedure requires relatively inexpensive equipment and technical expertise that are readily available in most clinical or animal research laboratory settings.

The method is validated by comparing the results obtained photographically to standard prism and cover assessments of macaque monkeys with strabismus. This comparison demonstrates that results obtained by the two methods are in good agreement and that the degree of accuracy is similar for the two methods. An estimate of the angle of deviation based on 10 photographs has a 95% confidence interval of about two degrees. Invest Ophthalmol Vis Sci 33:234-246, 1992

The ability to accurately and quantitatively assess the pattern of ocular misalignment is an important step in the process of diagnosing and understanding the causes of strabismus. Three common methods are used to assess eye alignment. The primary methods typically used in the clinical setting involve some version of prism and cover tests.1,2 An advantage of using these tests is that they can be performed quickly and inexpensively. A disadvantage is that they require extended cooperation of the subject, which can be difficult to obtain in young children or animals. These methods are too cumbersome and inefficient to easily permit a quantitative assessment of the misalignment state at more than a couple of fixation target locations (typically a “near” and a “distant” target in the primary position of gaze).

A second set of methods, which has been used primarily in eye movement research laboratory settings, relies on the use of magnetic eye coils3 or eye tracking devices.4,5 These methods can provide an extremely accurate measure of the eye alignment state in addition to providing dynamic measures of eye movements. A disadvantage of these procedures is their cost in terms of expensive equipment or technical expertise necessary for implementation, calibration, and maintenance. The equipment and technical personnel can be justified in an eye movement research laboratory, but are not generally available in other research or clinical settings where the primary objective is to obtain a quantitative assessment of binocular eye alignment.

A third procedure, called the Hirschberg test, is sometimes used in clinical settings for the assessment of strabismus, particularly in noncooperative patients.1,2 The clinician assesses misalignment of the eyes based upon the reflections off the corneas of a small point source of light. Historically, much confusion has surrounded the theoretical rationale and proper usage of this method for obtaining quantitative estimates of misalignment. However, explicit geometric models that quantify this procedure have been developed and verified.6,7

We recently extended the geometrical model of the Hirschberg test derived by Brodie7 and demonstrated that the method of using corneal light reflections can
be used to obtain information about the absolute magnitude of misalignment in addition to the measures of relative misalignment that he described. Specifically, we demonstrated that by combining corneal reflex information with estimates of the location of the centers of rotation of the eyes, equations for the visual axes of the two eyes can be specified. Then it is possible to determine the magnitude of horizontal alignment error of the visual axes from any fixation target in the observer's horizontal field of gaze. The final product of our extended model was a definition of three error terms that specify the discrepancy between the location of the fixation target and (1) the nearest approach of the visual axis for the left eye (Left_Error); (2) the nearest approach of the visual axis for the right eye (Right_Error); and (3) the location where the visual axes cross (Cross_Error).

Our previous model dealt only with eye alignment errors in the horizontal direction along the horizontal field of gaze. In this report, we extend our model in two ways. First, our model now allows eye position errors to be determined in the vertical as well as horizontal directions. Second, we are able to make these assessments in upper and lower gaze in addition to the horizontal meridian. Examples of misalignment patterns now distinguishable but not assessable with our previous methods include errors in vertical alignment, such as dissociated vertical deviations, and changes in horizontal deviations with changes in vertical gaze, such as A and V patterns.

There are a number of general advantages to our methodology.

1. It is inexpensive, requiring only items that are routinely available in most clinical or animal research settings.
2. It requires minimal technical expertise to set up, calibrate, and use.
3. It can be used with nonverbal subjects, including young children and animals.
4. A quantitative estimate of the alignment state at a particular target location can be obtained from a single momentary fixation even without extended cooperation from the subject.
5. Because of this increased efficiency, detailed information about fixation scatter during repeated tests at the same location and about patterns of misalignment to targets at a number of locations throughout the subject's field of gaze can be obtained.

Macaque monkeys were used as subjects to validate the method. However, our methods could be easily applied to other subject populations, including human children, in research laboratory or clinical settings.

**Materials and Methods**

**Subjects**

Nine Macaque monkeys (Macaca nemestrina) were used as subjects to validate the current model. These included two normal control monkeys and seven monkeys diagnosed by prism and cover test conducted during an ophthalmological examination as having some type of eye misalignment. All procedures with the monkey subjects were performed in strict compliance with the ARVO Resolution on the Use of Animals in Research.

**Assumptions of the Three Dimensional Model**

In our two-dimensional model, the shape of the eye was approximated as the union of two spheres, the globe of the eye, and the cornea. Specifically, we assumed that a slice through the eye in the horizontal plane that passes through the center of the pupil would be adequately described as the union of two circles. Based on these assumptions, the model's equations predicted the amount of rotation in the eye in the horizontal plane, independent of the shape of the surface in any other plane. All of the points necessary for the analysis, such as the centers of rotation of the eyes, the corneal centers of curvature, and the fixation targets, were assumed to be in this one optical plane. In three dimensions, this situation becomes more complicated because some of the necessary points fall outside of one plane.

In our three dimensional model, we have switched from a spherical to a toric surface to represent the cornea. We specify the radii of curvature of this surface in the vertical and horizontal planes, based on keratometry measurements obtained during an ophthalmological examination. Our approximation of corneal shape ignores other meridional variations and higher order aberrations. Another change from our previous two-dimensional model is that we use the line of sight to specify the axis for target fixation instead of the visual axis. This change makes no essential difference in the predictions of the model and has the advantage of allowing us to simplify some of the required calculations. We have retained from the two-dimensional model the assumptions that the eye rotates about its geometrical center of rotation and that the optical axis is coincident with the pupillary axis.

**The Model Eye.**

We specify locations in a Cartesian coordinate system, where the x,y,z position corresponding to 0,0,0 is located at the cyclopician center (Fig. 1). The x axis passes through the cyclopician center and through the center of rotation of each eye (R_e). The x,y plane,
Fig. 1. Schematic diagram of a typical photographic set-up. The subject's head is positioned such that the cyclopian center is located at (0, 0, 0), the center of rotation for the right eye $R_r$ is located at $(R_r, 0, 0)$, and for the left eye $R_l$ at $(R_l, 0, 0)$. Polarizing goggles are placed in front of the subject's eyes, and the polarity can be oriented crossed or uncrossed with respect to a large polarizing sheet located at the camera plane. This allows presentation of fixation targets to either or both eyes, while the camera records eye position from both eyes simultaneously. We typically present fixation targets at a number of positions in upper ($y = +30$ cm), middle ($y = 0$), and lower ($y = -30$) gaze fields.

where $z = 0$, defines the horizontal field of gaze. The straight ahead position for each eye is defined by $R_r, 0, 0$. Upward positions in the field of gaze are designated by $z > 0$, and downward positions by $z < 0$. Similarly, leftward positions are designated by $x > R$ and rightward by $x < R$.

In Figure 2 we illustrate the planar projections of a three-dimensional model eye onto horizontal $(y, x)$ and vertical $(y, z)$ planes. The center of the pupil (C) is defined as the bisection of the distance between the nasal (N) and temporal (T) limbus in the horizontal direction and its upper (U) and lower (L) limits in the vertical direction. The toric corneal surface produces two corneal centers of curvature in each eye, $B_h$ and $B_v$, for the horizontal and vertical curvatures, respectively. We assume that $R$, $B_h$, $B_v$, and C are collinear. This line defines the optical and the pupillary axes of the eye. The amount of rotation of the pupillary axis from straight ahead is given by the angle $\psi_{h,v}$. The angle between the pupillary axis and the line of sight, $\lambda_{h,v}$, can be estimated from monocular corneal light reflex photographs as the angle formed between the camera flash and the fixation target when the corneal reflex of the camera flash is centered in the pupil. We define the line of sight as the line that passes through point C and is rotated from straight ahead by the sum of angles $\psi$ and $\lambda$. We calculate error terms based on $\epsilon_{h,v}$, the deviation of the line of sight from the intended axis that passes through the fixation target.

Geometrical Estimation of the Hirschberg Ratio.

The geometrical optics underlying the Hirschberg ratio (the amount of rotation of the eye per millimeter displacement of the corneal light reflex from the center of the pupil) has been treated in depth previously. Briefly, as shown in Figure 2a for rotations in the horizontal plane, a perpendicular through the horizontal corneal center of curvature, $B_h$, intersects point $F_h$ in the film plane, the image of the corneal reflex in
Fig. 2. Planar projections of the three-dimensional model eye into the (A) horizontal (y, x) and (B) vertical (y, z) planes. The nasal (N) and temporal (T) extents of the limbus appear in the horizontal plane, and the upper (U) and lower (L) in the vertical plane. These projections produce two corneal centers of curvature $B_{h}$, $B_{v}$. We define the pupillary axis as the line passing through the center of rotation $R$, the corneal centers of curvature, and the center of the pupil $C$. The angle that the pupillary axis makes from straight ahead, $\psi_{h}$, can be determined from the amount that the corneal light reflex, $P_{h}$, is decentered from the photographic image of the pupillary center, $P_{c}$. The angle that the line of sight makes from straight ahead can be determined by finding angle $\lambda_{h}$. Thus, for a given fixation attempt, the angle made by the line of sight can be compared to the intended axis, that is, the angle of rotation that would be necessary for perfect fixation. The difference is the magnitude of the eye alignment error, $\epsilon_{h}$.

Photographic Methods

The physical layout when we make our photographs is illustrated in Figure 1. The subject’s head is positioned so that the cyclopian center is located at 0,0,0 and the left and right eye centers of rotation fall at $R_{l}$ and $R_{r}$. A camera (video or a standard 35 mm), equipped with a 55 mm lens and a ring flash that encircles the aperture of the lens, is positioned in front of the subject. Parallel to and at the distance of the camera lens, a sheet of gray polarizing material is placed, which extends approximately 50° outward from the camera. Two smaller pieces of polarizing material are placed in goggles worn by the subject. The individual polarizers in the goggles can be oriented crossed or uncrossed with respect to the large polarizing sheet. The polarizing sheets in crossed orientation block 98.3% transmission. These procedures provide a method for photographing both eyes simultaneously while presenting fixation targets that can be seen by either eye alone or by both eyes.

Fixation targets, consisting of small objects or food pellets held by forceps, are presented behind the polarizing sheet anywhere within the subject’s field of gaze. We present targets at a number of separate distance, lateral, and vertical locations. In a typical sequence, we try to sample from positions in upper, middle, and lower gaze from the planes illustrated in Figure 1.

A small millimeter ruler is placed above the eyes of each subject to be included in the photographs. This ruler serves as a millimeter reference for later corneal
light reflex measurements. Measurement of the corneal light reflex images can be done by projecting slides onto a screen if a 35 mm camera is used, or by grabbing the individual frames which captured the flash into a computerized graphics system if video is used. In either case, the distance of the corneal light reflex (in both the horizontal and vertical directions) is measured from the center of the pupil. In cases where one edge of the limbus is obscured by the lids, its position is estimated by extrapolating from the portion of the limbus that can be seen in the image.

Empirical Estimation of the Hirschberg Ratio

The geometrical model can be validated by measuring the Hirschberg ratio empirically. Monocular vision is induced by putting an occluder in the goggles or by using an occluder contact lens. Photographs are then taken while the subject views fixation targets at known eccentricities to each side of the camera for horizontal Hirschberg ratio measurements and above and below the camera for vertical measurements. From the photographs taken at each fixation angle, the amount of displacement of the corneal light reflex from the center of the pupil is measured, and plots of the fixation target angles (in degrees) versus the reflex displacements (in mm) for the horizontal and vertical directions are made. Straight lines can be fit to these data sets, and slopes of these lines yield empirical estimates of the horizontal and vertical Hirschberg ratios. Similarly, the displacement of the corneal light reflex from the pupillary center when the subject is induced to fixate the camera directly is an estimate of angle λ.

Once the Hirschberg ratio and angle λ have been determined for each eye using the methods just described, presenting fixation targets at known locations throughout the visual field and measuring binocular eye alignment errors are possible. We calculate five separate eye alignment error vectors for each fixation target: (1) horizontal Left_Error; (2) vertical Left_Error; (3) horizontal Right_Error; (4) vertical Right_Error; and (5) Cross_Error. Details about how we calculate these error vectors are presented in the Appendix. Magnitudes of the Left_Error and Right_Error vectors are expressed in degrees of rotation of the lines of sight from their intended target. Directions of these vectors are collapsed to a binary value reflecting eso- or exo-deviations in the case of horizontal error, and hyper- or hypo-deviations in the case of vertical error. Magnitude of Cross_Error is expressed in meter-angles of distance (1/distance in meters). This error term is derived as the difference between two other meter-angle distances: (1) the distance measured in the y direction between the observer plane (y = 0) and the fixation target; and (2) the distance in the y direction between the observer plane and the point where the vector projections of the lines of sight cross. The direction of Cross_Error is collapsed to a binary value that indicates eso- or exo-deviations.

Results

Geometric Determination of the Hirschberg Ratio

All monkeys in the study were given an ophthalmological examination under general anesthesia by a pediatric ophthalmologist to obtain the empirical measurements of corneal curvature and limbal radius. These values were used to calculate the predicted values of the Hirschberg ratio based upon Equation 1.0 for the horizontal and vertical directions for 8 monkeys (7 strabismic and 1 normal). The results are presented in Table 1.

The mean Hirschberg ratio across all monkeys was 14.0° of rotation per millimeter of corneal light reflex displacement for the horizontal direction, which agrees well with the same measure calculated previously based on our two-dimensional model on a smaller subset of these monkeys (14.1°/mm). For the vertical Hirschberg ratio, the mean was 14.2° of rotation per millimeter of corneal light reflex displacement. These Hirschberg values, regardless of direction, ranged from 13.0–16.0°/mm. The differences between the horizontal and vertical Hirschberg ratios within an individual eye usually were small, averaging about 6%. However, larger differences were found in individual cases, demonstrating that it is prudent to calculate separate horizontal and vertical values if keratometry measurements are available. Differences between the two eyes of the same monkey also were usually small, averaging about 3% in the horizontal and vertical directions. The Hirschberg ratios obtained for the normal monkey were not significantly different from those of the monkeys with strabismus.

Empirical Determination of the Hirschberg Ratio

The results from an empirical estimation of the horizontal and vertical Hirschberg ratios, based upon photographs taken while the monkeys were induced to take up monocular fixation at known eccentricities horizontally and vertically, are shown in Table 1. We plotted the fixation target angle versus the displacement of the corneal light reflex from the pupillary center and used linear regression to fit these data points with a straight line. In all cases, the goodness of fit of the data points to a straight line was over 90% for the horizontal and vertical directions. This indicates that, for the range of target eccentricities used (80°), a single value for the horizontal Hirschberg ratio and a single value for the vertical Hirschberg ratio are ap-
Table 1. Geometrical and empirical Hirschberg ratios (HR), and angle lambda derived for seven strabismic and one normal monkey

<table>
<thead>
<tr>
<th>Monkey</th>
<th>Eye</th>
<th>Horiz. geom. HR</th>
<th>Vert. geom. HR</th>
<th>Horiz. emp. HR</th>
<th>Vert. emp. HR</th>
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<th>Vert. angle lambda</th>
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appropriate. The mean of the empirical horizontal Hirschberg ratio across all monkeys was found to be 14.0° of rotation per millimeter of corneal light reflex displacement, the same as predicted by our geometrical model. Furthermore, the largest difference between the horizontal Hirschberg ratio predicted from the geometrical model and the empirical results was only 0.4°. Likewise, the mean of the empirical estimation of the vertical Hirschberg ratio was identical to the mean calculated from the model: 14.2°. The largest difference between the vertical Hirschberg ratio predicted from the geometrical model and the empirical results was only 0.6°. Once again, the values obtained for the normal monkey were indistinguishable from the values obtained for the strabismic monkeys.

Determination of Angle Lambda

The final values shown in Table 1 (derived photographically) are the horizontal and vertical angles $\lambda$. The mean angle $\lambda_h$ or the average amount the line of sight was displaced from the pupillary axis in the horizontal direction, was $+2.5^\circ$ for all monkeys. The positive sign indicates the line of sight intersects the retina of the eye on the nasal side of the pupillary axis. In other words, without accounting for angle $\lambda$, and using only the pupillary axis to determine eye alignment, an exodeviation would be improperly diagnosed when the eyes were actually properly fixating the target. The mean angle, $\lambda$, for the monkey subjects was $0.1^\circ$, indicating no significant displacement of the line of sight from the pupillary axis in the vertical direction.

Test-Retest Variability

Multiple photographs were obtained from normal control monkey PMH, whose results are presented in Table 1, and from a second control monkey, PBI, to determine intra- and inter-animal variability in these measurements. Five to 10 photographs were taken at each of the 36 separate fixation target locations, 12 each from the upper, middle, and lower gaze planes shown in Figure 1. Representative scatter data for control monkey PBI during repeated fixations at one target location (0,100,0) are shown in Figure 3. In this

Fig. 3. Eye alignment scatter plot for monkey PBI for fixation errors measured in the horizontal direction. The data represent the results obtained at one target location, straight ahead at 1 meter (0,100,0). Each R and L symbol represents a single fixation attempt during monocular left eye and right eye viewing, respectively. The amount of Right—Error for that fixation attempt is plotted on the ordinate, and the amount of Left—Error on the abscissa. Mean fixation error during binocular viewing is represented by the B symbol, and the error bars represent ± one standard deviation during binocular viewing conditions.
scatter plot, magnitude of Left—Error in the horizontal direction is presented on the abscissa, and magnitude of Right—Error on the ordinate. Each data point depicts the magnitude of horizontal error in the left and the right eye (regardless of which eye is viewing) during a single fixation attempt. Results from left eye viewing are shown by the L symbols, and from right eye viewing by the R symbols. Data points from both viewing (raw data not shown) also overlap with this same cluster.

We calculated the mean and standard deviations in the horizontal Left—Error and vertical Right—Error directions. The mean error for all three viewing conditions was near the (0,0) coordinate on the scatter plot, and the standard deviations all had a magnitude of about 1.5°. In Figure 3 we have superimposed the mean and standard deviation values for the binocular viewing condition with the raw data scatter values for left and right viewing. The data shown in this figure illustrate the results we obtained for all viewing conditions for both of our normal monkeys. No significant differences were found across any viewing condition or subset of fixation targets for either monkey. Furthermore, there were no significant differences between the two monkeys. Therefore, an average standard deviation was computed across all of the above conditions and across both monkeys. The values obtained for the standard deviation were 1.5° (for Left—Error and Right—Error) and 0.29 meter-angles (for Cross—Error). We interpret this to indicate that our ability to identify an eye position error in monoscopic monkeys using the corneal light reflex methodology from a single photograph is limited to errors greater than approximately 3° or 0.6 meter-angles. For an estimate based on 10 photographs, the 95% confidence interval of our mean estimate is about 2°, or 0.4 meter angles.

All of these assessments of trial-to-trial variability were done by having the same person score all of the photographs. A related question relates to inter-scorer variability when scoring is done by different persons.

To address this question, we took a sample of 20 consecutive images from one of our videotapes and had each image scored independently by two separate individuals. The mean difference between the two scorers in judged eye position was 0.7°.

Validation of the Corneal Light Reflex Methodology Against Traditional Prism and Cover Tests

In Table 2, we compare the corneal light reflex assessments of eye alignment to prism and cover assessments for the seven strabismic monkeys. The clinicians who conducted the cover tests were unaware of the results of the photographic methods at the time they made the assessments. Prism and cover estimates of the misalignment were made at near (approximately 33 cm) and at distance (approximately 2 m). A judgment was made at the same time regarding eye preference for fixation. Photographic measurements of the deviations were made to targets placed straight ahead at 33 cm (0,33,0) and 2.0 meters (0,200,0). Five to 10 photographs were taken at each location, and the result shown is the average. Results from the prism and cover tests were similar to those obtained by the photographic methods for every case in which a comparison could be made. In two cases, the photographic method produced estimates of the eye alignment state that could not be obtained by prism and cover because of the uncooperativeness of the monkeys.

Measuring Patterns of Binocular Alignment Errors by Photographic Methods

In this section, we present representative data from two strabismic monkeys to illustrate the types of information that can be revealed routinely using the photographic methodology. Our first example is a data set taken from monkey F84115. Clinical evaluation by a pediatric ophthalmologist using standard prism and cover tests diagnosed this monkey as having

<table>
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<th>Distance</th>
<th>Prism and cover assessment</th>
<th>Photographic assessment</th>
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* Unable to obtain accurate measurement.
measuring accommodative esotropia up to 15° at near and with no deviation at distance (see Table 2). We show the pattern of Right-Error, as assessed by the photographic methods, for deviations in the horizontal direction during binocular viewing of the fixation targets in Figure 4. The error surfaces shown in this figure illustrate how the magnitude of Right-Error varies with position across targets presented in upper, middle, and lower gaze corresponding to the planes shown in Figure 1.

The photographic corneal light reflex results shown in Figure 4 are consistent with the clinical diagnosis of an accommodative esotropia based on prism and cover. This shows up in Figure 4 as a pattern of increasing alignment error as fixation targets are placed nearer the monkey. Clinical evaluation found the magnitude of the deviation at near to be about 15°. This is confirmed by the height of the error surface at the 3 meter-angle distance during middle gaze. Similarly, the clinical finding of no deviation at distance is reflected by the height of the error surface at the 0.5 meter-angle viewing distance. However, the pattern of errors obtained from the photographic procedure also reveals additional information that was not appreciated based upon the clinical evaluation alone. For example, note in Figure 4 that no deviations are present for any fixation targets presented at distances of 1 meter-angle or farther. Also, note that the corneal light reflex procedure shows that the magnitude of Right-Error increases more in lower gaze, being approximately 20° at near, but is only about 13° for upper gaze at near.

A second example of this photographic method's utility is illustrated by a data set obtained from monkey M79434. Clinical evaluation diagnosed this monkey as having an alternating esotropia of 10-15°. We have presented data previously, based upon our two-dimensional model, that demonstrates the pattern of horizontal errors for this monkey is consistent with this clinical evaluation. Additional information obtained from this monkey with our three-dimensional model is shown in Figure 5. This data set shows the pattern of Right-Error magnitude for deviations in the vertical direction during monocular viewing of fixation targets with the left eye. These results demonstrate that this monkey exhibits a right hyperphoria during left eye fixation. This vertical deviation was not appreciated during cover and prism tests of this monkey.

Discussion

Clinical diagnoses of strabismus in human patients and assessments of binocular alignment in animals are necessarily limited by the accuracy of quantitative descriptions of the eye alignment error. In the present report, we have described and validated a photographic method based upon corneal light reflections for the quantitative assessment of ocular alignment. Our methods were developed using a geometrical model of the eye and verified empirically. Our proce-
Fig. 5. Eye position error surfaces for monkey M79434. All terms are the same as in Figure 3. The heights of the surfaces represent Right-Error magnitudes for deviations in the vertical direction during monocular viewing of fixation targets with the left eye. In this case, the pattern of errors reveals a right hyperphoria of approximately 6°.

The procedure allows for the measurement of eye alignment errors to fixation targets presented at any distance throughout the subject's field of gaze and provides a quantitative assessment of horizontal and vertical error. We have validated our method using clinical prism and cover methods. In addition, we have shown it to have some significant advantages. It can be easily applied to noncooperative subjects, such as animals and young children. The efficiency of the method makes it easy to obtain estimates of the scatter as well as magnitude of fixation error. Also, because of its increased efficiency, the method can provide information about patterns of errors across many fixation target locations. This pattern could be determined, in principle, with prism and cover tests. However, as a practical matter, this is not feasible, especially with noncooperative subjects. Furthermore, the procedure only requires relatively inexpensive equipment and technical expertise that is readily available in most clinical or animal research laboratory settings.

The major improvements of our three-dimensional method over our previous two-dimensional model are that it provides a way of measuring eye alignment for fixation targets placed throughout the subject's field of gaze and affords the ability to assess errors in both the horizontal and vertical directions. This is accomplished essentially by treating the horizontal eye position and the vertical eye position independently. That is, the Hirschberg ratio (the amount of rotation of the eye per millimeter of displacement of the corneal light reflex from the center of the pupil) is determined separately for rotations in the horizontal plane and in the vertical plane. This independence relies upon the assumption that the shape of the cornea is approximated adequately by a toric surface. The empirical validation of the results obtained by this method against standard prism and cover assessments provides evidence that this assumption is reasonable.

The average Hirschberg ratio across the monkeys studied was approximately 14° of rotation per millimeter of corneal light reflex displacement. This result is comparable to the values found in our previous study based upon the two-dimensional model, which examined only horizontal errors in a subset of these same monkeys. The predicted Hirschberg ratio from the geometrical model and the Hirschberg ratio determined empirically never differed by more than one-half degree. Differences were sometimes found between the horizontal Hirschberg ratio and the vertical Hirschberg ratio within the eye of an individual monkey, demonstrating the benefit of using a model based on a toric instead of a spherical corneal surface. Furthermore, plots of the data used in the empirical measurement of the Hirschberg ratio were fit adequately by straight lines over the ±40° range measured. Therefore, at least over this range of eccentricities measured, one value is adequate for specifying the horizontal Hirschberg ratio, and one value is adequate for the vertical Hirschberg ratio.

Some of the other changes between our two-dimensional model and our current three-dimensional model also are worth noting. For example, the line of sight is now used instead of the visual axis. This simplifies the model because a single line of sight can be defined that is applicable for vertical and horizontal measurements. Separate horizontal and vertical visual axes would be required as these are constrained to pass through the separate vertical and horizontal corneal centers of curvature. This switch to the line of sight makes little or no practical difference in terms of...
the results obtained. The magnitude of the difference is specified by the angular separation of a line that passes from a given point in the visual field through the center of the pupil (for the line of sight) compared to a line that passes from the same point in space through the horizontal or vertical corneal center of curvature (for the visual axis). Because the distance between the center of the pupil of the eye and the corneal center of curvature is on the scale of millimeters, the angle subtended by intersecting lines passing through those two points will be negligible.

The magnitude of Left_Error and Right_Error error terms in the horizontal direction depends upon the interocular distance. Removing this dependency is possible by restating these error terms into their cyclopian equivalents. We have shown previously that

$$\varepsilon_{oc} = \tan^{-1} \frac{x \tan \varepsilon_h}{x \pm r}$$

where $\varepsilon_{oc}$ is the magnitude of the deviation in the horizontal direction for a cyclopian-centered eye. It can be derived by knowing the horizontal component of the nearest approach of the line of sight, $x$, the interocular distance, $2r$, and the horizontal error magnitude, $\varepsilon_h$. Left_Error and Right_Error remain the same for a cyclopian eye. The expression of alignment error in terms of these cyclopian values is advantageous in that it allows our specification of misalignment to be related directly to Hering's basic law of innervation. When specified in relation to a cyclopian eye, our error terms reduce to three values corresponding to magnitude of the error along the horizontal ($x$), vertical ($z$), and distance ($y$) axes. Hering argued that the direction of the discrepancy between the present and the intended point of sight is grasped by the oculomotor system in the form of deviations in three planes, which can be directly related to our three cyclopian error terms. Hering's basic law of innervation asserts that elimination of the deviation specified by these three error terms is accomplished by the innervation of three corresponding muscle groups.

Our error terms specify the positions of the lines of sight from the two eyes in three-dimensional space but do not provide any direct information about the horopter, ie, whether or not off-axis targets fall on corresponding points on the two eyes. A related limitation is that we have no way to measure cyclorotations of the eyes around the lines of sight. Thus, we have no way to evaluate deviations from Listing's law. In principle, this information could be incorporated by using the methods previously described by Nakayama.

Similarly, another limitation of our methods is that they provide no information about dynamic properties of eye movements. We determine only the static alignment state during target fixation. Some questions about strabismus can be addressed only by measuring the dynamic properties. In these cases eye coil or eye tracking methods remain the methods of choice for measurements of eye position. However, a number of important questions about ocular alignment that arise frequently in clinical or animal research laboratory settings can be adequately addressed with our photographic methods.

The scatter plots of our data, as illustrated in Figure 3, reveal that trial-to-trial variability in eye position for normal monkeys as judged by our methods has a standard deviation of about 1.5°. This limits our ability to detect eye position errors to those larger than a couple of degrees even if we base our estimate of eye position on the mean taken from 10 separate photographs. Similar variability probably will be present when these methods are applied to other subject populations, such as human infants. This level of accuracy is roughly comparable to prism and cover tests, but is about an order of magnitude poorer in resolution than magnetic search coil methods or the best eye tracking systems.

An issue that warrants further discussion is whether this variability could be reduced to increase the accuracy of our method. The scatter in our data comes from three sources: (1) experimental variability during data collection; (2) experimental variability during data scoring; and (3) observer fixation error. We have determined that the primary source of error during data collection relates to uncertainty of head position at the time the photograph is taken. To illustrate the influence of small fluctuations in head position, note that a head displacement of 1 cm from our fixed cyclopian center will induce approximately 1° of apparent error for near target fixation. In the studies reported in this paper, we did not hold the head rigidly in place, and we estimate that head position varied as much as 1 cm from trial to trial. Thus, we could increase the accuracy of our photographic methods substantially if we were to implement a way of holding the head more immobile or compensating for its movements, as is done with more sophisticated eye tracking systems. However, our goal has been to develop a method that is applicable for use in noninvasive animal studies or with human infants and that can be implemented cheaply and without extensive technical expertise or extensive training of personnel for carrying out the procedures. If accuracy finer than a couple of degrees is needed, our belief is that switching to an eye coil or one of the more sophisticated eye tracking methodologies probably would be more efficient than putting substantial effort into further refining this relatively simple procedure.

Similarly, a number of experimental parameters
can be manipulated in an attempt to reduce the amount of methodological variability introduced during scoring. We have found that inter-scorer differences, when scoring the same photograph, usually fall in the range of about 0.5–1.0° when naive scorers are used and receive no feedback or cross checking about how their results compare with other scorers. This can be reduced to less than 0.5° if scorers are given training or feedback about how their results compare to a standard scorer. Other factors that can be manipulated include trying to find the optimal amount of magnification of the image during scoring and attempting to increase the image resolution in pixels when scoring from video. We have manipulated these variables in an informal way and found them to have only marginal influence as long as they are kept within a reasonable range. For example, a naive assumption might be that resolution will be limited by pixel resolution in the video display. However, we have found that in practice this factor makes relatively little difference, presumably for the same reasons that vernier acuity is affected little by receptor spacing.14

The third source of scatter is the most interesting from a theoretical perspective. It arises from inherent variability on the part of the observer when attempts are being made to direct its eyes toward a particular target. Traditional measures of binocular alignment have been concerned only with the magnitude of the error in strabismus and provide no information about issues of scatter. The increased efficiency of our photographic method allows repeated measures to be obtained from the same fixation target location to generate scatter plots of the type illustrated in Figure 3. Examination of the clusters formed by these scatter plots has the potential to provide additional information about characteristics of strabismus. In a normal individual, the clusters for left eye, right eye, and binocular viewing are similar to one another in three ways: (1) the center of each cluster falls near the (0,0) coordinate value; (2) the clusters all form a shape that is approximately circular; and (3) the diameters of the circular clusters are all similar. In some of our strabismic monkeys, we have found abnormalities in these properties of the scatter in addition to the errors in mean amount of convergence. These findings will be elaborated on in a future report.

Appendix

Step 1. Determine the horizontal and vertical rotation of the line of sight: In our description of the model in the text, we described rotations of the eye with reference to the y-axis because this is the easiest to understand with reference to the film plane (Fig. 2). However, we find the calculations to be simpler if we specify rotations in the horizontal plane as the angle formed by the line of sight with the x-axis, which we designate at α. By convention, we assume that when the eye is in the primary position α = 90. Similarly, for calculating rotations in the vertical plane, we specify the angle of the line of sight from the positive z-axis, where β = 90 when the eye is in primary position. When an eye moves to fixate a target, this movement can be described as a rotation α in the x-y plane and a rotation β in the z-y plane. If we think of these two planar rotations as occurring independently, two orthogonal angles of rotation for the line of sight can be derived. The equation for the horizontal component, derived from Equation 1.0 (see text) and including angle λ is:

$$\alpha = 90 + \sin^{-1} \left( \frac{P_x}{P_y} \right) + \lambda$$ (1.1h)

That is, we have calculated the rotation of the line of sight as if it were occurring only in the x-y plane. We then can repeat this for the rotation in the z-y plane, this time using the comparable vertical parameters:

$$\beta = 90 + \sin^{-1} \left( \frac{P_z}{P_y} \right) + \lambda$$ (1.1v)

The end result yields two orthogonal angles of rotation for the line of sight.

Step 2. Determine the vector from the cyclopian center to the center of rotation: Consider the cyclopean center to be the origin of a vector R that extends to the center of rotation of one eye. Methods for estimating the position of the geometrical center of rotation have been described previously.8 The magnitude of this vector is R, and its direction is solely along the x axis:

$$R = R_xI$$ (2.0)

Step 3. Determine the vector that extends from the center of rotation to the center of the pupil: We will label this vector RC. Its magnitude is simply the distance from R to C. Its horizontal direction can be found from the angle θ, given:

$$90 + \psi = \alpha + \lambda$$ (3.0h)

where angles α and λ were determined from Equation 1.1h. The vertical direction of vector RC can be found from the angle ψ, given:

$$90 + \psi = \beta + \lambda$$ (3.0v)

where angles β and λ were determined from Equation 1.1v. The vector RC can be defined as:

$$RC = RC_x + RC_y + RC_z$$ (3.1)

where the vector component in the x direction is:

$$RC_x = RC \cos(\alpha + \lambda)$$ (3.2)
the vector component in the y direction is:
\[ RC_y = RC_{xy} \sin(\alpha_s + \lambda_s) \]  
(3.3)
and the vector component in the z direction is:
\[ RC_z = RC \cos(\beta_s + \lambda_s) \]  
(3.4)

The vector \( RC_{xy} \) is the projection into the x-y plane of the original vector \( RC \).

**Step 4. Through vector addition, define the vector from the cyclopian center to the center of the pupil:** This step is accomplished by adding the vectors described by Equations 2.0 and 3.1, yielding the desired vector \( C \).

\[ C = (R_x + RC_x)i + RC_{yj} + RC_{zk} \]  
(4.0)

**Step 5. Define a vector from the cyclopian center to the fixation target:** The next step is to find the vector \( F \) that passes from the cyclopian center to the point in space that defines the fixation target.

\[ F = F_xi + F_yj + F_zk \]  
(5.0)

where the direction components are the Cartesian coordinates of the fixation target.

**Step 6. Find the vector from the center of the pupil to the fixation target:** From the previous step, we found the vector \( F \) from the cyclopian center to the fixation target. From Step 4 (Equation 4.0), we found the vector \( C \) from the cyclopian center to the center of the pupil. If we simplify the notation of this vector:

\[ C = C_xi + C_yj + C_zk \]  
(6.0)

then the vector that extends from the center of the pupil to the fixation target can be found by vector subtraction as:

\[ CF = (C_x - F_x)i + (C_y - F_y)j + (C_z - F_z)k \]  
(6.1)

This vector defines the intended axis (Fig. 2). It corresponds to the line of sight in a perfectly aligned eye.

**Step 7. Find the magnitude and direction of the eye position error:** Recall that from the corneal light reflex measurements of our observer (Step 1), we have already described the angle \( \alpha_s \) that the observer’s line of sight makes away from the x axis in the x-y plane and the angle \( \beta_s \) that the observer’s line of sight makes away from the z-axis in the z-y plane. To determine the magnitude of the position error of our observer’s line of sight, all that remains is to find the corresponding angles made by the vector \( CF \) and the x and z axes, respectively, and compare them to the angles found by the corneal light reflex method.

If the previous vector equation for the intended axis (Equation 6.1) is simplified:

\[ CF = CF_xi + CF_yj + CF_zk \]  
(7.0)

then the angle this vector makes with the x axis is:

\[ \alpha_p = \tan^{-1} \frac{CF_y}{CF_x} \]  
(7.1h)

and the angle that this vector makes with the z axis is:

\[ \beta_p = \tan^{-1} \frac{CF_z}{CF_x} \]  
(7.1v)

Angles \( \alpha_p \) and \( \beta_p \) are the horizontal and vertical rotations of the line of sight that would be required for accurate fixation. Therefore, the magnitude of the eye position error in the horizontal direction, \( \alpha_m \), is the absolute value of the difference between the values found in Equations 1.0h and 7.1h:

\[ \alpha_m = |\alpha_p - \alpha_s| \]  
(7.2h)

Likewise, the magnitude of the eye position error in the vertical direction, \( \beta_m \), is:

\[ \beta_m = |\beta_p - \beta_s| \]  
(7.2v)

The direction of the eye alignment error can be determined from the sign of \( \beta_m \) and \( \alpha_m \) found in the previous two equations. For both eyes, a negative \( \beta_m \) indicates hypodeviations, and a positive value indicates hyperdeviations. To determine the direction of the horizontal error, the eye being measured also must be known. For Left_Error, eso-deviations are indicated when \( \alpha_m \) is positive. For Right_Error, eso-deviations are indicated when \( \alpha_m \) is negative.

The above steps provide a method for determining Left_Error and Right_Error in three dimensions. What remains is to find the comparable values for Cross_Error.

**Step 8. Define a vector from the cyclopian center to the "point of intersection" of the lines of sight from the two eyes:** In our two-dimensional model, we defined Cross_Error in reference to the location in space where the visual axes from the two eyes crossed. However, in the three-dimensional model, the lines representing the lines of sight from the two eyes are not necessarily coplanar, and it is possible they fail to intersect. Thus, in our current model, we estimate the nearest approach of the lines of sight from the two eyes we denote as I, the “point of intersection.” Once that point is identified, a vector in three dimensions from the cyclopian center to I can be determined.

Using the x-y plane as the example, we can solve for the intersection point in the horizontal plane. We need the x and y values in rectangular coordinates for

\* The quotation marks around “point of intersection” indicate that point I is an estimate of the point at which the lines of sight come closest to intersecting. The phrase “intersection point,” without quotation marks, will be used to indicate an actual point of crossing of lines projected onto a single plane.
points $C_{lh}$ and $C_{rh}$, the centers of the pupils for the left and right eyes, respectively, and the slopes of the lines of sight of the two eyes. From Equation 1.1h, we can obtain the angles $\alpha_l$ and $\alpha_r$, representing the amount of rotation of the lines of sight for the left and right eyes in the x-y plane. The slope of the lines corresponding to these angles can be determined by taking the tangent of these angles.

To obtain the rectangular coordinates of the centers of the pupils of the two eyes, we can use the horizontal component of vector C. The component vector $C_{lh}$ from Equation 6.0 for the left eye, would be:

$$C_{lh} = C_{xh}i + C_{yh}j$$  \hspace{1cm} (8.0)

Because the origin of vector $C_{lh}$ is the cyclopian center, the rectangular coordinates for point $C_{lh}$ are simply $(C_x, C_y)$. Similarly, we can find the comparable coordinates for vector $C_{rh}$. With this information, the point-slope form of the equations for the projections of the lines of sight from the two eyes can be derived. These equations can be equated and the intersection point solved.9

By projecting the lines of sight and the vectors $C_l$ and $C_r$ into the vertical plane, the z coordinate for point I can be found using the same procedure. Because we are projecting into the x-y and z-y planes, we obtain two estimates of the y coordinate of the “point of intersection.” Therefore, we take the mean of the two values as the estimate of the y coordinate. Once the rectangular coordinates of point I are determined, the vector from the cyclopian center to the “point of intersection” can be put in vector notation:

$$I = I_xi + I_yj + I_k$$  \hspace{1cm} (8.1)

**Step 9. Find the magnitude and direction of Cross—Error:** We express the magnitude of the eye position error as the absolute value of the difference between the y axis distance of the cyclopian center to the fixation target (expressed in meter-angles), and the y axis distance of the cyclopian center to the “point of intersection” (also expressed in meter-angles). The distance $F_y$ from the cyclopian center to the fixation target can be obtained from equation 5.0. This distance in meter-angles is:

$$F_m = \frac{1}{F_y}$$  \hspace{1cm} (9.0)

and the comparable measure $I_y$ to the “point of intersection” is found in equation 8.1. This distance in meter-angles is:

$$I_m = \frac{1}{I_y}$$  \hspace{1cm} (9.1)

The magnitude of the Cross—Error, $m$, is given by the absolute value of:

$$m = I_m - F_m$$  \hspace{1cm} (9.2)

The direction of the eye alignment error can be determined by the sign of the value $m$. If the value is positive, the “point of intersection” is nearer the subject than the fixation target, indicating an eso-deviation. A negative value indicates an exodeviation.

**Key words:** ocular alignment, corneal light reflex, Hirschberg test, strabismus, pediatric ophthalmology, monkey

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### References