Quantitative Evaluation of Irregular Astigmatism by Fourier Series Harmonic Analysis of Videokeratography Data

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PURPOSE. To assess quantitatively corneal irregular astigmatism in association with best spectacle-corrected visual acuity.

METHODS. Refractive powers on a mire ring measured with computerized videokeratography were decomposed, using the Fourier series harmonic analysis. Extracting spherical and regular astigmatic components, the remaining irregular astigmatic component was quantified on rings 2 through 9. A weighted average was calculated by using the Stiles-Crawford effect on the basis of the radius of each ring of each eye and was used as an index of the irregular astigmatic component. Data analyses were carried out in 108 eyes, including 53 normal eyes, 34 eyes with keratoconus, and 21 eyes that had undergone penetrating keratoplasty for keratoconus. Keratoconic eyes and eyes after keratoplasty were included in the study only if visual acuity, corrected with a hard contact lens, was 20/20 or better. Logarithm of best spectacle-corrected visual acuity, age, type of disease, refractive astigmatism, irregular astigmatic component, surface regularity index, and surface asymmetry index were analyzed.

RESULTS. In results of multiple regression analysis, the irregular astigmatic component was significantly correlated with best spectacle-corrected visual acuity \( r = -0.744; \) adjusted \( R^2 = 0.549; P < 0.001 \), whereas other explanatory variables showed no correlation with best spectacle-corrected visual acuity.

CONCLUSIONS. This model of the irregular astigmatic component seems to be an efficient, quantitative means of describing corneal irregular astigmatism. (Invest Ophthalmol Vis Sci. 1998;39:705-709)

Refractive powers of the cornea can be represented by three principal components: spherical power, regular astigmatic power, and irregular astigmatic power.1,2 Among these, spherical and regular astigmatic components can be corrected by a spherocylindrical lens, whereas irregular astigmatism cannot be corrected by a toric spectacle lens.2-4

Irregular astigmatism may be created by such surgical procedures as cataract surgery, penetrating keratoplasty, refractive surgery, and pterygium removal surgery. Corneal diseases and trauma are sources of irregular astigmatism. In these situations, visual acuity with spectacle correction remains poor, whereas the best-corrected visual acuity using contact lenses may be normal. Patients with a high degree of irregular astigmatism would be dissatisfied with the results of an otherwise successful procedure or treatment. Therefore, precise and reproducible evaluation of irregular astigmatism is of clinical importance and of investigational interest.

Recently, Hjortdal et al.5 and Raasch6 described a method for evaluating the optical information of videokeratographic data using the Fourier series harmonic analysis. These investigators applied the method in normal eyes,5,6 in eyes with corneal diseases,5 and in eyes after ocular surgery5,7 and demonstrated its ability to separate quantitatively spherical power, regular astigmatism and irregular astigmatism.5-7 Theoretically, the amount of irregular astigmatism is inversely correlated with spectacle-corrected visual acuity,6 but such an association has not been examined. In the present study, we evaluated the correlation between videokeratographically calculated irregular astigmatism and visual acuity corrected with spectacles in eyes with normal and pathologic histories.

MATERIALS AND METHODS

Subjects

One hundred and eight eyes of 72 subjects were evaluated, including 53 eyes of 27 normal subjects, 34 eyes of 24 patients with keratoconus, and 21 eyes of 21 patients who had undergone penetrating keratoplasty for keratoconus. Eyes in the normal control group had no ocular disease except for mild refractive errors, including myopia, hyperopia, and regular forms of astigmatism (less than 1.5 diopters [DI]). The best-corrected visual acuity with a spherocylindrical lens was 20/20 or better in each case. None of the subjects was wearing

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Figure 1. Dioptric power distribution of ring 3 obtained from a keratoconic cornea (original power). Spherical equivalent corresponds to the first term of the Fourier equation (equation 2).

By using a Fourier series harmonic analysis, dioptric powers on a mire ring \( i \), \( F_i(\sigma) \), can be transformed into trigonimic components of the following form:

\[
F_i(\sigma) = a_0 + \sum [a_n \cos(n\sigma) + b_n \sin(n\sigma)]
\]  

(1)

This can be written as

\[
F_i(\sigma) = a_0 + c_1 \cos(\sigma - \alpha_1) + c_2 \cos 2(\sigma - \alpha_2) + \cdots + c_n \cos n(\sigma - \alpha_n)
\]

(2)

where \( a_0 \) is the spherical equivalent of the ring, \( c_i \) the skewness or decentration component, \( c_2 \) the regular astigmatic component, \( \alpha_2 \) the phase (or axis) of regular astigmatism, and \( c_3 \ldots n \) the nonregular astigmatic components in a strict sense (Fig. 1, 2). Among these, spherical equivalent power \( (a_0) \) and regular astigmatism \((\text{second harmonic component;} n = 2)\) can be corrected by a sphericylindrical lens, whereas the remaining components \((n = 1 \text{ and } n \geq 3)\) cannot. Therefore, corneal irregular astigmatism in a broad sense, \( L_i \), can be represented by excluding spherical \( (a_0) \) and regular astigmatic \( (c_2) \) components from the measured dioptric power distribution. This can be achieved by using the least-squares method to fit \( F_i(\sigma) \) to a sinusoidal curve comprising \( a_0 \) and \( c_2 \) components, and by calculating the final loss of the approximation (root mean square of \( c_1 \) and \( c_3 \ldots n \) astigmatism).

\[
L_i = \frac{\sum [F_i(\sigma) - (a_0 + c_2 \cos 2(\sigma - \alpha_2))]}{3}
\]

(3)

where \( n \) is the number of available measurement points on a ring.

These calculations were performed on rings 2 through 9, which approximately represent the central 3-mm zone of the cornea, confirmed in 111 normal eyes (Table 1). The distance between the videokeratographic corneal center and measurement point \( j \) \((1 \leq j \leq 256)\) of ring \( i \), \( d(j) \), was averaged to obtain the mean radius of the ring, \( r_i \), for each eye. Thereafter, the weighted average of 8 rings \((2 \leq i \leq 9)\) was calculated using the Stiles-Crawford effect \((\text{e}^{-0.105r^2})\).

\[
r_i = \frac{\sum d(j)}{n}
\]

(4)

\[
\text{IR} = \sum \text{e}^{-0.105r^2} \times L_i/8
\]

(5)

Equation 4 was applied for each ring of each eye, and the calculated values were used in equation 5. The obtained regular astigmatism component \((\text{IR})\) was used as an index of the irregular astigmatic component of each eye.
TABLE 1. Average Radius of Mire Rings in Normal Subjects

<table>
<thead>
<tr>
<th>Ring Number</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.221 ± 0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.398 ± 0.058</td>
</tr>
<tr>
<td>3</td>
<td>0.560 ± 0.027</td>
</tr>
<tr>
<td>4</td>
<td>0.728 ± 0.043</td>
</tr>
<tr>
<td>5</td>
<td>0.903 ± 0.030</td>
</tr>
<tr>
<td>6</td>
<td>1.068 ± 0.036</td>
</tr>
<tr>
<td>7</td>
<td>1.241 ± 0.041</td>
</tr>
<tr>
<td>8</td>
<td>1.411 ± 0.046</td>
</tr>
<tr>
<td>9</td>
<td>1.580 ± 0.051</td>
</tr>
<tr>
<td>10</td>
<td>1.748 ± 0.056</td>
</tr>
<tr>
<td>11</td>
<td>1.918 ± 0.061</td>
</tr>
<tr>
<td>12</td>
<td>2.089 ± 0.066</td>
</tr>
<tr>
<td>13</td>
<td>2.257 ± 0.072</td>
</tr>
<tr>
<td>14</td>
<td>2.429 ± 0.077</td>
</tr>
<tr>
<td>15</td>
<td>2.601 ± 0.083</td>
</tr>
<tr>
<td>16</td>
<td>2.774 ± 0.089</td>
</tr>
<tr>
<td>17</td>
<td>2.949 ± 0.095</td>
</tr>
<tr>
<td>18</td>
<td>3.125 ± 0.100</td>
</tr>
<tr>
<td>19</td>
<td>3.304 ± 0.107</td>
</tr>
<tr>
<td>20</td>
<td>3.486 ± 0.114</td>
</tr>
<tr>
<td>21</td>
<td>3.670 ± 0.120</td>
</tr>
<tr>
<td>22</td>
<td>3.857 ± 0.127</td>
</tr>
<tr>
<td>23</td>
<td>4.045 ± 0.137</td>
</tr>
<tr>
<td>24</td>
<td>4.236 ± 0.146</td>
</tr>
<tr>
<td>25</td>
<td>4.435 ± 0.157</td>
</tr>
</tbody>
</table>

Values are mean ± standard deviation; n = 111 eyes.

Results

Patient demographics are summarized in Table 2. Results of multiple linear regression analysis showed a significant negative correlation between the irregular astigmatism component and the logarithm of best spectacle-corrected visual acuity ($r = -0.744; P < 0.001$).

DISCUSSION

Transformation of periodic curves into Fourier harmonic components is widely used in electrical and biomedical engineering. In the present study, the nonspherocylindrical corneal power component was extracted from the videokeratography data, by Fourier series harmonic analysis, and a weighted average was calculated on rings 2 through 9. The data of ring 1 were excluded from the analysis because of the potential inaccuracy of data acquisition at the innermost ring. The remaining variables did not show statistically significant contributions (Table 3). The logarithm of best spectacle-corrected visual acuity was predicted by $-0.122 \times IR + 0.0163$, with an adjusted $R^2$ value of 0.549. Other methods used to select explanatory variables, such as forward-selection, the stepwise technique, or the maximum $R^2$ improvement technique produced similar results, showing that variables other than the irregular astigmatism component were minimally related to the logarithm of best spectacle-corrected visual acuity.

The type of disease was coded: 0, 0, normal control eyes; 0, 1, keratoconic eyes; and 1, 0, eyes after keratoplasty. The backward-elimination technique was used to select explanatory variables having a statistically significant contribution to the logarithm of best spectacle-corrected visual acuity.

Example of a graph showing the relationship between irregular astigmatic component and logarithm of best spectacle-corrected visual acuity.

FIGURE 3. Irregular astigmatic component and logarithm of best spectacle-corrected visual acuity. Results of multiple regression analysis showed a significant correlation, ($r = -0.744; P < 0.001$).
because some of the 256 data points are often absent in peripheral rings, especially in eyes with pathologic conditions. In the present study including keratoconic and transplanted eyes, complete rings were obtained to include ring 9 in all cases. Moreover, there was a significant positive correlation between irregular astigmatic components calculated on 24 rings (r = 0.970; P < 0.001), and we verified that multiple regression analysis in which values were used on 24 rings produced similar results.

For the weighted average calculation, we used the Stiles-Crawford effect and the radius of each mire ring in each eye. However, the axis of videokeratography does not correspond exactly to the line of sight, to the pupil’s axis, or to the geometric axis of the cornea. The distance on the cornea between the pupil’s axis (reference axis of the Stiles-Crawford effect) and the videokeratographic axis depends on the angle κ (angle between the line of sight and the pupil’s axis), the corneal radius of curvature, and the anterior chamber’s depth.5 In practice, it is difficult to consider all parameters in individual eyes. Moreover, determination of the reference point of a videokeratography system itself depends on several topologic approximations, on accurate fixation by the patient, and on proper alignment of the instrument. 18 Given these limitations, the calculation methods used in the present study appear to fulfill the reasonable requirements of a clinically convenient evaluation method for irregular astigmatism.

As shown in the results of multiple linear regression analysis, the irregular astigmatic component was the single factor predictive of best spectacle-corrected visual acuity. Other parameters, including refractive astigmatism, surface regularity index, and surface asymmetry index, did not contribute to best spectacle-corrected visual acuity. We did not include keratometric cylinders in the analysis because keratometer measurements were impossible in several keratoconic and transplanted eyes. However, when those eyes were excluded and multiple regression analysis was performed on 96 eyes in which keratometer readings were acquired, similar results were obtained.

The surface regularity index and the surface asymmetry index are useful quantitative descriptors of the corneal contour, and both have been shown to correlate with best spectacle-corrected visual acuity.13,14 In the present patients, the surface regularity and surface asymmetry indexes had significant sample correlation coefficients with the logarithm of best spectacle-corrected visual acuity (r = −0.618; P < 0.001 and r = −0.263; P < 0.01, respectively). Nevertheless, when partial correlation coefficients controlling for irregular astigmatic component were calculated, values were not statistically significant (r = −0.1415; P = 0.152 and r = 0.1342; P = 0.174, respectively). Conversely, the partial correlation coefficient between the logarithm of best spectacle-corrected visual acuity and irregular astigmatic component, when controlling for surface regularity and surface asymmetry indexes, remained statistically significant (r = −0.6218; P < 0.001).

In videokeratography, more samples are commonly collected per unit area from the central cornea than from the periphery.19 To remove bias from the imbalance of the area represented by a measurement point from different rings, an area compensation technique has been used in the calculation of several videokeratography-derived indexes.19 If obtained values in equation 3 were counterbalanced based on the area represented by each ring—that is, the area within that ring subtracted by that within the neighboring inner ring—the correlation coefficient between the irregular astigmatic component and best spectacle-corrected visual acuity would be r = −0.723 (adjusted R² = 0.521, P < 0.001). Although overall results of the analysis were similar, the prediction became marginally less accurate than that for the original multiple regression equation (adjusted R² = 0.549). At present, we have no explanation for this discrepancy, and the necessity of area-based correction cannot be determined in the data analysis that deals with the powers in a circumferential fashion along the mire, as was performed in the present study. Further studies will be needed to clarify this point.

With the accelerated interest in refractive corneal surgery and enhanced refinement of intraocular surgery, there has been a widening realization that quantitative measurements of corneal contour have significance in evaluating the effects of these procedures. The methods presented in this report allow mathematical separation of regular and irregular astigmatic components, and may be useful in the assessment of the optics of the eye in which irregular astigmatism is of interest.

References


